Orthogonal Frequency Division Multiplexing for Wireless Networks

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***Abstract*— When a high data rate is to be transmitted over a channel with relatively large maximum delay an alternative approach is given by the OFDM transmission technique. OFDM is a promising technique for the broadband wireless communication system. The idea of OFDM is to distribute the high rate data stream to many low rate data streams that are transmitted in a parallel way over many sub-channels, thus in each sub-channel the symbol duration is low as compared to maximum delay of channel and ISI can be handled, but it is up to some extent. However, a special problem in OFDM is its vulnerability to frequency offset errors due to which the orthogonality is destroyed that result in ICI. ICI causes power leakage among subcarriers thus degrading the system performance. In this paper we described the adaptive equalization for multipath faded OFDM system. Generally in OFDM system a rectangular pulse shaping technique is applied, the ICI will not occur, however out of band interference will present. Also we observed the Gaussian function for pulse shaping; in this case, the sub carrier spectra are also Gaussian functions and delay rapidly. In such a system, however both ISI and ICI over a few modulation symbols occur. The adaptive equalization with LMS and RLS algorithm mitigates this interference.**

Keywords*— Orthogonal frequency Division Multiplexing (OFDM); Inter Symbol Interference (ISI); Inter Carrier Interference (ICI).

Introduction

OFDM is emerging as the preferred modulation scheme in modern high data rate wireless communication systems. OFDM has been adopted in the European digital audio and video broadcast radio system and is being investigated for broadband indoor wireless communications. OFDM is a special case of multi-carrier modulation. Multi-carrier modulation is the concept of splitting a signal into a number of signals, modulating each of these new signals to several frequency channels, and combining the data received on the multiple channels at the receiver. In OFDM, the multiple frequency channels, known as sub-carriers, are orthogonal to each other. The separation of the subcarriers is theoretically minimal such that there is a very compact spectral utilization. These subcarriers have different frequencies and they are orthogonal to each other [3]. Since the bandwidth is narrower, each sub channel requires a longer symbol period. Due to the increased symbol duration, the ISI over each channel is reduced.

But problem in OFDM is its susceptibility to frequency offset errors between the transmitted and received signals, which may be caused by Doppler shift in the channel or by the difference between the transmitter and receiver local oscillator frequencies [4]. In such situations, the orthogonality of the carriers is no longer maintained, which results in Intercarrier Interference (ICI).

ICI results from the other sub-channels in the same data block of the same user. ICI problem would become more complicated when the multipath fading is present [5]. If ICI is not properly compensated it results in power leakage among the subcarriers, thus degrading the system performance.

One of the principal advantages of OFDM is its utility for transmission at very nearly optimum performance in unequalized channels and in multipath channels.

2. Data transmission using multiple carriers

An OFDM signal consists of a sum of subcarriers that are modulated by using phase shift keying (PSK) or qudrature amplitude modulation (QAM). If d_i are the complex QAM symbol, N_s is the number of subcarriers, T the symbol duration, and $f_i = f_0 + i/T$ the carrier frequency, then one OFDM symbol
starting at $t = t_s$ can be written as:

$$s(t) = Re\left\{\sum_{i=0}^{N_s-1} d_i \exp\left(j2\pi f_i (t - t_s)\right)\right\} \qquad (1)$$

$$s(t) = 0, \qquad t < t_s \ \wedge t > t_s + T$$

In the literature, often the equivalent complex notation is used, which is given by (2). In this representation, the real and imaginary parts correspond to the in-phase and quadrature parts of the OFDM signal, which have to be multiplied by a cosine and sine of the desired carrier frequency to produce the final OFDM signal. Figure (1) shows the operation of the OFDM modular in block diagram.

$$s(t) = \sum_{i=0}^{N_S-1} d_i \exp(j2\pi f_i(t - t_s)), t_s \le t \le t_s + T \qquad (2)$$

$$s(t) = 0, \qquad t < t_s \ \wedge t > t_s + T$$

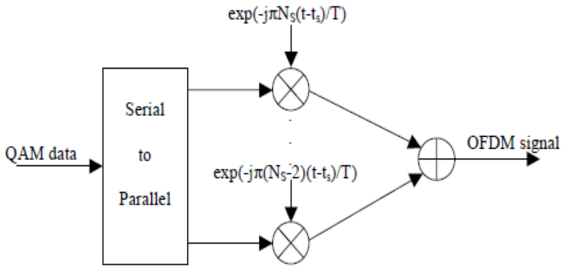

Fig.1. OFDM Modulator

As an example , figure (2) shows four subcarriers from one OFDM signal. In this example, all subcarriersn have the phase and amplitude, but in practice the amplitudes and phases may be modulated differently for each subcarrier. Note that each subcarrier has exactly an integer number of cycles in the interval ***T*** , and the number of cycles between adjacent subcarries differs by exactly one. This properly accounts for the orthogoality between subcarriers.

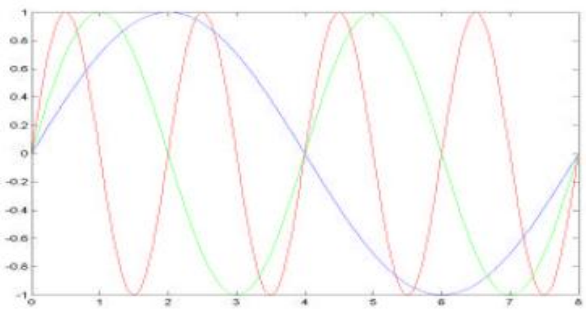

Figure 2

For instance, if the ***jth*** subcarrier from (2) is demodulated by down converting the signal with a frequency of $f_j = f_0 + j/T$ and then integrating the signal over ***T*** seconds, the result is as written in (3). By looking at the intermediate result, it can be seen that a complex carrier is integrated over ***T*** seconds. For the demodulated subcarrier *j*, this integration gives the desired output d_j (multiplied by a constant factor ***T***), which is the QAM value for that particular subcarriers. For all other subcarriers , this integration is zero, because the frequency difference $(i - j)/T$ produce an integer number of cycles within the integration interval T ,such that the integration result is always zero.

$$\int_{t_s}^{t_s+T} \exp\left(-2j\pi f_j(t - t_s)\right) \sum_{i=0}^{N_S-1} d_i \exp(j2\pi f_i(t - t_s))dt \qquad (3)$$

$$= \sum_{i=0}^{N_S-1} d_i \int_{t_s}^{t_s+T} \exp\left(-2j\pi \frac{i-j}{T}(t - t_s)\right) dt = d_j T$$

The orthogonality of different OFDM subcarriers can also be demonstrated in another way. According to (1), each OFDM symbol contains subcarriers that are nonzero over a T - seconds interval. Hence, the spectrum of a single symbol is a convolution of group of Dirac pulses located at the subcarrier frequencies with the spectrum of a square pulse that is one for a Tsecond period and zero otherwise. The amplitude spectrum of the square pulse is equal to sinc(πfT) , which has zeros for all frequencies f that are an integer multiple of 1/T . This effect is shown in figure which shows the overlapping sinc spectra of individual subcarriers. At the maximum of each subcarrier spectrum, all other subcarrier spectra are zero. Because an OFDM receiver calculates the spectrum values at those points that correspond to the maxima of individual subcarrier, it can demodulate each subcarrier free from any interference from the other subcarriers. Basically, Figure (3) shows that the OFDM spectrum fulfills Nyquist's criterion for an inter symbol interference free pulse shape. Notice that the pulse shape is present in frequency domain and
note in the time domain, for which the Nyquist criterion usually is applied. Therefore, instead of intersymbol interference (ISI), it is intercarrier interference (ICI) that avoided by having the maximum of one subcarrier spectrum correspond to zero crossing of all the others.

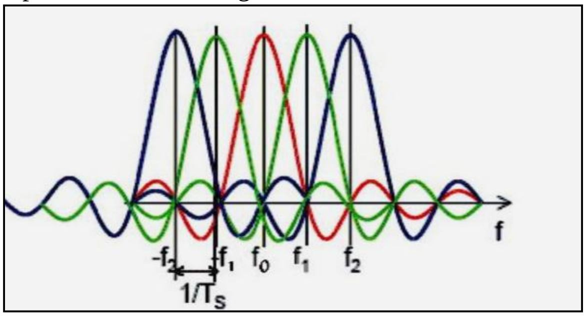

Figure 3

3. Generation of subcarriers using the IFFT

The complex baseband OFDM signal as defined by (2) is in fact nothing more than the inverse Fourier transform of S N QAM input symbol. The time discrete equivalent is the inverse discrete Fourier transform (IDFT), which is given by:

$$s(n) = \sum_{i=0}^{N_S-1} d_i \exp(j2\pi \frac{in}{N}) \qquad (4)$$

Where the time t is replaced by a sample number n. In practice, this transform can be implemented very efficiently by the inverse Fast Fourier transform (IFFT) as shown in figure(4) and (5).

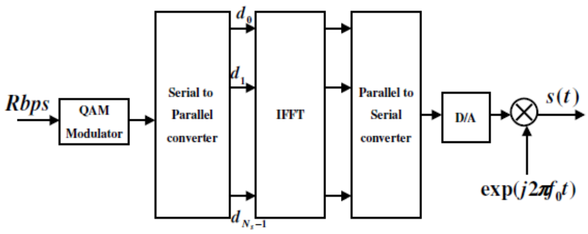

Figure 4 Transmitter

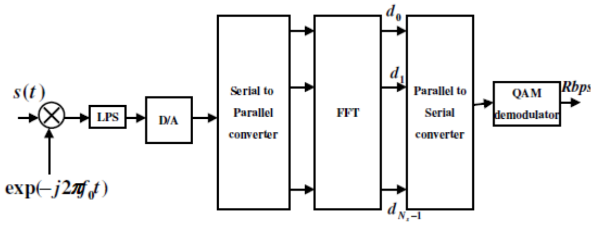

Figure 5 Receiver

4. Guard time and cyclic extension

One of the most important reasons to do OFDM is the efficient way it deals with multipath delay spread. By dividing the input data stream in ***s N*** subcarriers, the symbol duration is made ***s N*** times smaller, which also reduces the relative multipath delay spread, relative to symbol time, by the same factor. To eliminate intersymbol interference almost completely, a guard time is introduced for each OFDM symbol. The guard time is chosen larger than the expected delay spread, such that multipath components from one symbol cannot interfere with the next symbol. The guard time could consist of no signal at all. In that case, however, the problem of intercarrier (ICI) would arise. ICI is crosstalk between different subcarriers,which means they are no longer orthogonal. This effect is illustrated in figure (6) in this example, a subcarrier 1 and a delayed subcarrier 2 are shown. When an OFDM receiver tries to demodulate the first subcarrier, it will encounter some interference from the second subcarrier, because within the FFT interval, there is no integer number of cycles difference between subcarrier 1and 2. At the same time, there will be crosstalk from the first to the second subcarrier for the same reason.

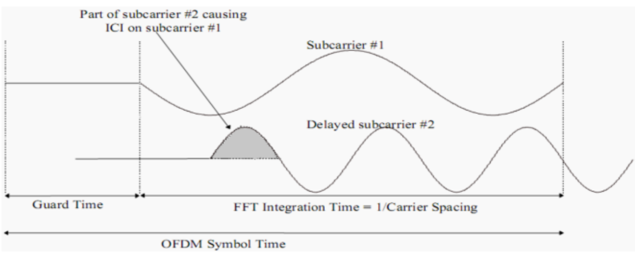

Figure 6

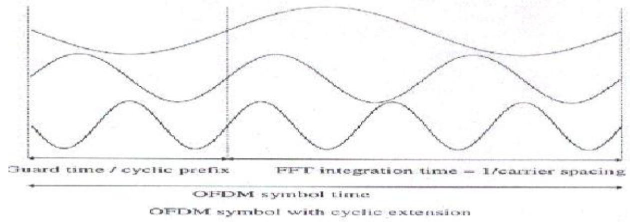

Figure 7

To eliminate ICI, the OFDM symbol is cyclically extended in the guard time, as shown in figure(7). This ensures that delayed replicas of the OFDM symbol always have an integer number of cycles within the FFT interval, as long as the delay is smaller than the guard time. As result, multipath signals with delays smaller than the guard time cannot cause ICI.

As an example of how multipath effects OFDM, figure(8) shows received signal for tow-ray channel, where the dotted curve is a delayed replica of the solid curve. Three separate subcarriers are shown during three symbol intervals. In reality, an OFDM receiver only sees the sum of all these signals, but showing the separate components makes it more clear what the effect of multipath is. From the figure, we can see that the OFDM subcarriers are BPSK modulated,which means that there can be 180-degree phase jumps at the symbol boundaries. For the dotted curve, these phase jumps occur at a certain delay after the first path. In this particular example, this multipath delay is smaller than the guard time, which means there are no phase transition during the FFT interval. Hence, an OFDM receiver "sees" the sum of pure sine waves with some phase offsets. This summation does not destroy the orthogonality between the subcarries, it only introduces a different phase shift for each subcarrier. The orthogonality does become lost if the multipath delay becomes larger than the guard time. In that case, the phase transitions of delayed path fall within the FFT interval of the receiver. The summation of the sine waves of the first path with the phase modulated waves of the delayed path no longer gives a set of orthogonal pure sine waves, resulting in
a certain level of interference.

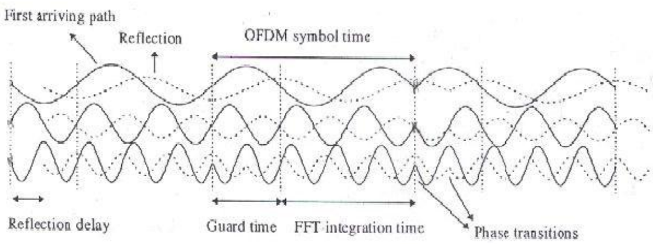

Figure 8

To get an idea what level of interference is introduced when the multipath delay exceeds the guard time, Figure (9) depicts three constellation diagrams that were drived from a simulation of an OFDM link with 48 subcarriers, each modulated by using 16-QAM. Figure (9)a shows the undistorted 16-QAM constellation, which is observed whenever the multipath delay is below the guard time. In figure (9)b, the multipath delay exceeds the guard time by a small 3% fraction of the FFT interval. Hence, the subcarriers are not orthogonal any more but the interference is still small enough to get a reasonable received constellation. In Figure (9)c, the multipath delay exceeds the guard time by 10% of the FFT interval.

The interference is now so large that the constellation is seriously blurred, causing an unacceptable error rate.

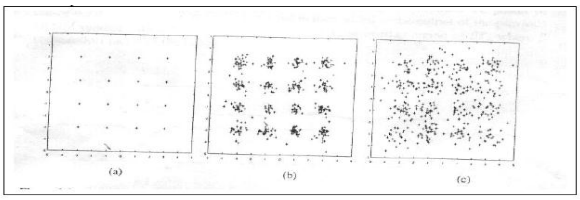

Figure 9

5. windowing

In the previous sections, it was explained how an OFDM symbol is formed by performing an IFFT and adding a cyclic extension. Looking at an example OFDM signal like in figure (8), sharp phase transition caused by the modulation can be seen at the symbol boundaries. Essentially, an OFDM signal like the one depicted in figure (8) consists of a number of unfiltered QAM subcarriers. As a result, the out-of-band spectrum decreases rather slowly, according to a sinc function. As an example of this, the spectra for 16, 64, and 256 subcarriers are plotted in Figure (10). For larger number of subcarriers, the spectrum the spectrum goes down rapidly in the beginning, which is caused by the fact that the sidelobes are closer together. However, even the spectrum for 256 subcarriers has relatively large –40 dB bandwidth that is almost four times the – 3 dB bandwidth.

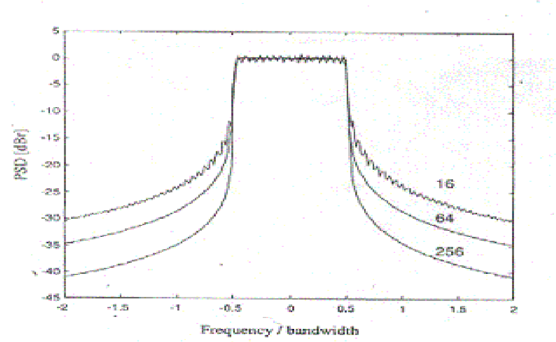

Figure 10

To make the spectrum goes down more rapidly, windowing can be applied to the individual OFDM symbol. Windowing an OFDM symbol makes the amplitude go smoothly to zero at the symbol boundaries. A commonly used window type is the raised cosine window. Here, ***s T*** is the symbol interval, which is shorter than the total symbol duration because we allow adjacent symbols to partially overlap in the roll-of region.

In equation form, an OFDM symbol starting at time $t = t_s = kT_s$ is defined as:

$$s(t) = Re\left\{w(t-t_s)\sum_{i=0}^{N_s-1} d_i \exp\left(j2\pi f_i\left(t-t_s - T_{prefix}\right)\right)\right\},$$

$$t_s \le t \le t_s + T_s(1+\beta)$$

$$s_k(t) = 0, t < t_s \wedge t > t_s + T_s(1+\beta)$$

where β is the roll-off factor of the raised cosine

In practice, the OFDM signal is generated as follows: first, ***s N*** input QAM values are padded with zeros to get ***N*** input samples that are used to calculate an IFFT. Then, the last T_{perfix} samples of the IFFT output are inserted at the start of the OFDM symbol, and the first $T_{postfix}$ samples are appended at the end. The OFDM symbol is then multiplied by a raised cosine window ***w(t)*** to more quickly reduce the power of out-of-band subcarriers. The OFDM symbol is then added to the output of the previous OFDM symbol with a delay of T_s , such that there is an overlap region of βT_s , where b is the roll-off factor of the raised cosine window. Instead of windowing, it is also possible to use convolutional filtering techniques to reduce the effect of the out-of-band spectrum.

6. Choice of OFDM parameters

The choice of various OFDM parameters is a tradeoff between various, often conflicting requirements. Usually, there are three main requirements to start with: bandwidth, bit rate, and delay spread. The delay spread directly dictates the guard time. As a rule, the guard time should be about two to four times the root-mean-squared delay spread. This value depends on the type of coding and QAM modulation. Higher order QAM (like 64-QAM) is more sensitive to ICI and ISI than QPSK; while heavier coding obviously reduces the sensitivity to such interference. Now the guard time has been set, the symbol duration can be fixed. To minimize the signal-to-noise ratio (SNR) loss caused by guard time, it is desirable to have the symbol duration much larger than the guard time. It cannot be arbitrarily large, however, because a larger symbol duration means more subcarriers with a smaller subcarrier spacing, a larger implementation complexity, and more sensitivity to phase noise and frequency offset, as well as an increased peak-to-average power ratio. Hence, a practical design choice to make the symbol duration at least five times the guard time, which implies a 1dB SNR loss because the guard time. After the symbol duration and guard time are fixed, the number of subcarriers follows directly as the required –3 dB bandwidth divided by the subcarrier spacing,which is the inverse of the symbol duration less the guard time. Alternatively, the number of subcarriers may be determined by the required bit rate divided by the bit rate per subcarrier. The bit rate per subcarrier is defined by the modulation type, coding rate, and symbol rate. An additional requirement that can affect the chosen parameters is the demand for an integer number of samples both within the FFT/IFFT interval and in the symbol interval.

7. Basic OFDM Model

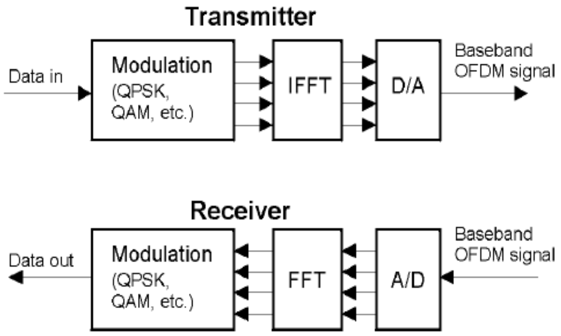

Fig.11 Basic FFT,OFDM Transmitter & receiver

8. Conclusion

There is some debate as to whether multicarrier or single carrier modulation is better for ISI channels with delay spreads on the order of the symbol time. It is claimed in that for some mobile radio applications , single carrier with equalization has roughly the same performance as multicarrier modulation with channel coding, frequency-domain interleaving, and weighted maximum-likelihood decoding. Adaptive loading was not taken into account , which has the potential to significantly improve multicarrier . But there are other problems with multicarrier modulation that impair its performance, most significantly frequency offset and timing jitter, which degrade the orthogonality of the subchannels. In addition, the peak-to-average power ratio of multi carrier is significantly higher than that of single carrier systems, which is a serious problem when nonlinear amplifiers are used . Tradeoffs between multicarrier and single carrier block transmission systems with respect to these impairments are discussed. Despite these challenges, multicarrier techniques are common in high data rate wireless systems with moderate to large delay spread, as they have significant advantages over time-domain equalization. In particular, the number of taps

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