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Orthogonal Frequency Division Multiplexing & Channel Estimation

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***Abstract* - Orthogonal Frequency Division Multiplexing is advantageous digital communication technique used in wireless applications because of high data rate transmission capability and high bandwidth efficiency. The channel estimation techniques for OFDM system based on pilot arrangement. The channel estimation based on comb type pilot arrangement and Block type pilot arrangement. LS, MMSE, Modified MMSE, LS estimator with 1D interpolation and other estimators are discussed in this survey paper. Comparison of performance of various estimators is given.**

I. INTRODUCTION

OFDM is becoming widely applied in wireless communications systems due to its high rate transmission capability with high bandwidth efficiency and its robustness with regard to multi-path fading and delay. It has been used in digital audio broadcasting (DAB) systems, digital video broadcasting (DVB) systems, digital subscriber line (DSL) standards, and wireless LAN standards such as WiFi and its European equivalent HIPRLAN/2. It has also been proposed for wireless broadband access standards such as WiMAX and as the core technique for the fourth-generation (4G) wireless mobile communications.

The use of differential phase-shift keying (DPSK) in OFDM systems avoids need to track a time varying channel; however, it limits the number of bits per symbol and results in a 3 dB loss in signal-to-noise ratio (SNR). Coherent modulation allows arbitrary signal constellations, but efficient channel estimation strategies are required for coherent detection and decoding.

The main problems in designing channel estimators for wireless OFDM system the arrangement of pilot information, where pilot means the reference signal used by both transmitters and receivers and the design of an estimator with both low complexity and good channel tracking ability. The two problems are interconnected.
In general, the fading channel of OFDM systems can be viewed as a two-dimensional (2D) signal (time and frequency). The optimal channel estimator in terms of mean-square error is based on 2D Wiener filter interpolation. Unfortunately, such a 2D estimator structure is too complex for practical implementation. The combination of high data rates and low bit error rates in OFDM systems necessitates the use of estimators that have both low complexity and high accuracy, where the two constraints work against each other and a good trade-off is needed. The one-dimensional (1D) channel estimations are usually adopted in OFDM systems to accomplish the trade-off between complexity and accuracy. The two basic 1D channel estimations are block-type pilot channel estimation and comb-type pilot channel estimation, in which the pilots are inserted in the frequency direction and in the time direction, respectively. The estimations for the block-type pilot arrangement can be based on least square (LS), minimum mean-square error (MMSE), and modified MMSE. The estimations for the comb-type pilot arrangement includes the LS estimator with 1D interpolation, the maximum likelihood (ML) estimator, and the parametric channel modeling-based (PCMB) estimator. Other channel estimation strategies are such as the estimators based on simplified 2D interpolations, the estimators based on iterative filtering and decoding, estimators for the OFDM systems with multiple transmit-and-receive antennas, and so on.

II. SYSTEM DESCRIPTION

The basic idea underlying OFDM systems is the division of the available frequency spectrum into several subcarriers. To obtain a high spectral efficiency, the frequency responses of the subcarriers are overlapping and orthogonal, hence the name OFDM. This orthogonality can be completely maintained with a small price in a loss in SNR, even though the signal passes through a time dispersive fading channel, by introducing a cyclic prefix (CP). A block diagram of a baseband OFDM system is shown in Figure 2.1

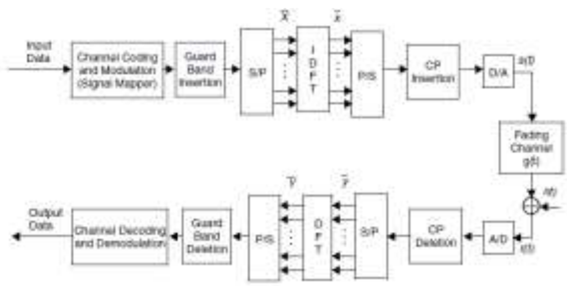


Fig. 2.1: A Digital Implementation of a Baseband OFDM System

The binary information is first grouped, coded, and mapped according to the modulation in a "signal mapper." After the guard band is inserted, an N-point inverse discrete-time Fourier transform (IDFTN) block transforms the data sequence into time domain (note that N is typically 256 or larger). Following the IDFT block, a cyclic extension of time length TG, chosen to be larger than the expected delay spread, is inserted to avoid intersymbol and intercarrier interferences. The D/A converter contains low-pass filters with bandwidth 1/TS, where TS is the sampling interval. The channel is modeled as an impulse response g(t) followed by the complex additive white Gaussian noise (AWGN) n(t).

At the receiver, after passing through the analog-to-digital converter (ADC) and removing the CP, the DFTN is used to transform the data back to frequency domain. Lastly, the binary information data is obtained back after the demodulation and channel decoding.

III. Channel Estimation

As shown in figure x_k are transmitted symbols, g(t) is the channel impulse response, $\tilde{n}$(t) is white complex Gaussian channel noise and y_k are the received symbols. The D/A and A/D converters contain ideal low pass filters with bandwidth $1/T_s$ where T_s is the sampling interval. A cyclic extension of time length T_{G} is used to eliminate inter-block interference and to preserve orthogonality of the tones.

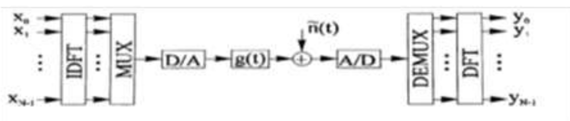


Fig. 3.1 Baseband OFDM System

The channel impulse response g(t) as a time limited pulse train is given by,

$$g(t) = \sum_m \alpha\delta(t - \tau mTs) \quad (1)$$

where αm is a complex values and $0 \leq \tau m Ts \leq$ TG.

The system is then modeled using N point discrete time Fourier Transform as,

$$Y = DFT(IDFT_N(x) \odot \frac{g}{\sqrt{N}}) + \tilde{n} \quad (2)$$

For n- input signals and corresponding n-output signals parallel Gaussian channels are as shown below,

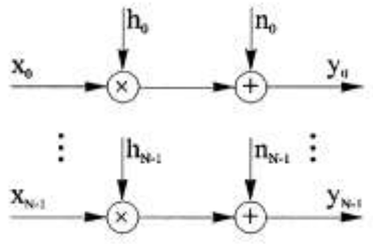


Fig. 3.2 Parallel Gaussian Channels

For a set of N independent Gausian channels,

$$y_k = h_k x_k + n_k \qquad k=0, \ldots N-1 \quad (3)$$

Using matrix notation we can write (3) as,

$$y = X F_g + n \quad (4)$$

where X is the matrix with the elements of x on its diagonal and

$$F = \begin{bmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \quad (5)$$

Where,

$$W_N^{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi nk/N} \quad (6)$$

The general channel estimator schematic is as shown below,

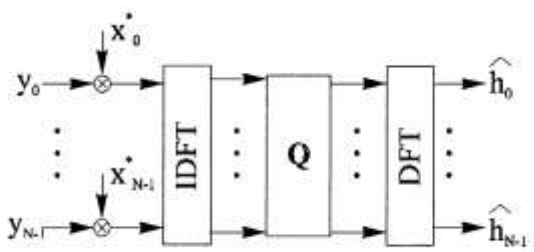


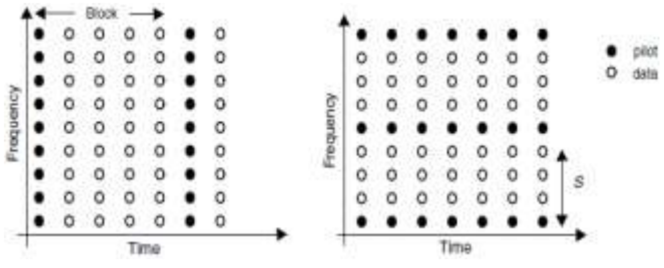
Fig. 3.3: General Estimator Structure

As a result, the fading channel of the OFDM system can be viewed as a 2D lattice in a time-frequency plane, which is sampled at pilot positions and the channel characteristics between pilots are estimated by interpolation. The art in designing channel estimators is to solve this problem with a good trade-off between complexity and performance.

IV. Types Of Channel Estimation

There are two main types of techniques for channel estimation and frequency offset estimation. One is pilot-aided, the other is blind. Pilot-aided method can track a rapid-varying channel at the cost of the transmission efficiency. So when the channel is slowly varying, such as the channel of the indoor transmission or the channel in which the mobiles move in a slow speed, a blind method can be used.

The two basic 1D channel estimations in OFDM systems are illustrated in Figure 4.1. The first one, block-type pilot channel estimation, is developed under the assumption of slow fading channel, and it is performed by inserting pilot tones into all subcarriers of OFDM symbols within a specific period. The second one, comb-type pilot channel estimation, is introduced to satisfy the need for equalizing when the channel changes even from one OFDM block to the subsequent one. It is thus performed by inserting pilot tones into certain subcarriers of each OFDM symbol, where the interpolation is needed to estimate the conditions of data subcarriers.



(a)Block-Type Pilot Channel Estimation **(b)Comb**-Type Pilot Channel Estimation

Fig. 4.1: Two Basic Types of Pilot Arrangement for OFDM Channel Estimations

4. Block-Type Pilot Channel Estimation:

In block-type pilot-based channel estimation, as shown in Figure 4.1 (a), OFDM channel estimation symbols are transmitted periodically, and all subcarriers are used as pilots. The task here is to estimate the channel conditions (specified by $\bar{H}$ or $\bar{g}$) given the pilot signals (specified by matrix X or vector $\overline{X}$) and received signals (specified by $\bar{Y}$), with or without using certain knowledge of the channel statistics. The receiver uses the estimated channel conditions to decode the received data inside the block until the next pilot symbol arrives. The estimation can be based on least square (LS), minimum mean-square error (MMSE), and modified MMSE.

4.1.1. MMSE AND LS ESTIMATORS:

If the channel vector g is Gaussian and uncorrelated with the channel noise n, then MMSE estimate of g becomes,

$$\hat{g}\text{MMSE} = R_{gy} R_{yy}^{-1} \tag{7}$$

Where,

$R_{gy} = E\{gy^H\} = R_{gg}F^H X^H$

$R_{yy} = E\{yy^H\} = XF\, R_{gg}F^H X^H + \sigma_n^2 I_N$

are cross covariance matrix between g and y and auto covariance matrix of y. Further Rgg is the auto covariance matrix of g and σ_n^2 denotes noise variance E $\{|n_k|^2\}$. These two quantities assumed to be known. Since the columns in the F are orthogonal, $\hat{g}_{MMSE}$ generates the frequency domain MMSE estimate, $\hat{h}^{MMSE}$ by,

$$\hat{h}_{MMSE} = \hat{g}_{MMSE} = FQ_{MMSE}F^H X^H \tag{8}$$

Where,

$$Q_{MMSE} = Rgg[(F^H X^H XF)^{-1}\sigma_n^2 + R_{gg}]^{-1}(F^H X^H XF)^{-1} \tag{9}$$

The LS estimator for Cyclic impulse response g minimizes $(y- XF_g)^H (y-XF_g)$ and generates,

$$\hat{h}_{LS} = FQ_{LS}F^H X^H y \tag{10}$$

Where,

$$Q_{LS} = (F^H X^H XF)^{-1} \tag{11}$$

And $\hat{h}_{LS}$ reduces to,

$$\hat{h}_{LS} = X^{-1}y \tag{12}$$

Both LS and MMSE esimator have drawbacks,MMSE esimator suffers from a high complexity and whereas the Ls estimator has high mean square error.

4.1.2. MODIFIED MMSE ESTIMATOR:

The MMSE estimator requires the calculation of an N X N matrix Q_{MMSE}, which implies a high complexity when N is large. Astraightforward way of decreasing the complexity is to reduce the size of Q_{MMSE}. We know that most of the energy in g is contained in, or near, the first L = $[T_G/T_S]$ taps. Thetefore we study a modification of the MMSE estimator, where only the taps with significant energy are considered. The elements in the RGG corresponding to low energy taps in g are approximated by zero. If we take into account the first L taps of g, and set R_{GG} (r,s) = 0 for r, s € [0,L-1],then Q_{MMSE} effectively reduced to an L X L martix. If T denotes the first Lcolumns of the DFT- matrix F and R'_{gg} denotes the upper left L X L corner of R_{gg}, then the modified MMSE estimator becomes,

$$\hat{h}_{MMSE} = TQ'_{MMSE} \tag{13}$$

Where

$$Q'_{MMSE} = [R'_{gg}[T^H X^H XT]^{-1}\sigma_n^2 + R'_{gg}]^{-1}(T^H X^H T)^{-1} \tag{14}$$

Thus the complexity of the MMSE estimator decrease considerably.

Estimation with Decision Feedback:

In block-type pilot-based channels, the estimators are usually calculated once per block and are used until the next pilot symbol arrives. The channel estimation with decision feedback is proposed to improve the performance, where the estimators inside the block are updated using the decision feedback equalizer at each subcarrier. The receiver first estimates the channel conditions using the pilots and obtains $\hat{H} = \{\hat{H}_k\}$ (k = 0, ….,N-1), which is based on LS, MMSE, or modified MMSE. Inside the block, for each coming symbol and for its each subcarrier, the estimated transmitted signal is found by the previous $\hat{H}_k$ according to the formula $\hat{X}_k = Y_k / \hat{H}_k$.{ $\hat{X}$ } is mapped to the binary data through the demodulation according to the "signal demapper," and then obtained back though "signal mapper" as .{$\check{X}_k$ }. The estimated channel H_k is updated by $H_k = Y_k / \check{X}_k$ and is used in the next symbol.

4.2 *Comb-Type Pilot Channel Estimation:*

In comb-type pilot based channel estimation, as shown in **Figure 2**, for each transmitted symbol, *Np* pilot signals are uniformly inserted into $\bar{X}$ with *S* with subcarriers apart from each other, where $S = N/N_p$.

4.2.1. LS Estimator with 1D Interpolation:

1D interpolation is used to estimate the channel at data subcarriers, where the vector $(\hat{H}_p)_{LS}$ with length *Np* is interpolated to the vector $\hat{H}$ with length *N*, without using additional knowledge of the channel statistics. The 1D interpolation methods are summarized in the remainder of this section.

4.2.1.1. Linear Interpolation (LI):

The LI method performs better than the piecewise-constant interpolation, where the channel estimation at the data subcarrier between two pilot H_{LS}^p(k) and H_{LS}^p (k+1) is given by,

$$\hat{H}(kS+t) = H_{LS}^p(k) + H_{LS}^p(k+1) - H_{LS}^p(k)(t/S) \quad (0 \le t \le S)$$

4.2.1.2. Second-Order Interpolation (SOI):

The SOI method performs better than the LI method, where the channel estimation at the data subcarrier is obtained by weighted linear combination of the three adjacent pilot estimates.

4.2.1.3. Low-Pass Interpolation (LPI):

The LPI method is performed by inserting zeros into the original $\hat{H}^P_{LS}$ sequence and then applying a low-pass finite-length impulse response (FIR) filter (the *interp* function in MATLAB), which allows the original data to pass through unchanged. This method also interpolates such that the mean-square error between the interpolated points and their ideal values is minimized.

4.2.1.4. Spline Cubic Interpolation (SCI):

The SCI method produces a smooth and continuous polynomial fitted to given data points (the *spline* function in MATLAB).

4.2.1.5. Time Domain Interpolation (TDI):

The TDI method is a high-resolution interpolation based on zero-padding and DFT/IDFT. It first converts $\hat{H}_{LS}^P$ to time domain by IDFT and then interpolate the time domain sequence to *N* points with simple piecewise-constant method. Finally, the DFT converts the interpolated time domain sequence back to the frequency domain.

In the performance among these estimation techniques usually ranges from the best to the worst, as follows: LPI, SCI, TDI, SOI, and LI. Also, LPI and SCI yield almost the same best performance in the low and middle SNR scenarios, while LPI outperforms SCI at the high SNR scenario. In terms of the complexity, TDI, LPI and SCI have roughly the same computational burden, while SOI and LI have less complexity. As a result, LPI and SCI are usually recommended because they yield the best trade-off between performance and complexity.

4.3. Other Pilot-Aided Channel Estimations:

4.3.1. Simplified 2D Estimators:

In 2D channel estimation, the pilots are inserted in both the time and frequency domains, and the estimators are based on 2D filters. In general, 2D channel estimation yields better performance than the 1D scheme, at the expense of higher computational complexity and processing delay. The optimal solution in terms of mean-square error is based on 2D Wiener filter interpolation, which employs the second-order statistics of the channel conditions. However, such a 2D estimator structure suffers from a huge computational complexity, especially when the DFT size *N* is several hundred or larger. A proposed algorithm with two concatenated 1D linear interpolations on frequency and time sequentially minimizes the system complexity. I, channel estimators based on 2D least square (LS) and 2D normalized least square (NLS) are proposed, and a parallel 2D (N)LS channel estimation scheme solves the realization problem due to the high computational complexity of 2D adaptive channel estimation.

4.3.2 Iterative Channel Estimators:

To reduce complexity, the 2D transmission lattice is divided by 2D blocks, and the pilots are uniformly inserted inside each block. Channel estimation proceeds on a block-by-block basis. The first estimator is based on iterative filtering and decoding, which consists of two cascaded 1D Wiener filters to interpolate the unknown time-varying 2D frequency response between the known pilot symbols. The second estimator uses an *a posteriori* probability (APP) algorithm, in which the two APP estimators, one for the frequency and the other for the time direction, are embedded in an iterative loop similar to the turbo decoding principle. These iterative estimators yield robust performance even at low SNR scenarios, but with high computation complexity and certain iteration time delay.

2) *Channel Estimators for OFDM with Multiple Antennas:* Multiple transmit-and-receive antennas in OFDM systems can improve communication quality and capacity. For the OFDM systems with multiple transmit antennas, each tone at each receiver antenna is associated with multiple channel parameters, which makes channel estimation difficult. Fortunately, channel parameters for different tones of each channel are correlated and the channel estimators are based on this correlation.

V. PERFORMANCE EVALUATION

In general, 1D channel estimation schemes have a much lower computational complexity than 2D schemes because they avoid computing 2D matrices. Also, block-type pilot-channel estimation schemes are usually simpler than comb-type pilot schemes because they calculate the estimators once per block. In the block-type pilot schemes with decision feedback, the estimators are updated for each symbol by simple vector division. Algorithm complexity, ranking from low to high, is summarized in Table for the block-type pilot arrangement.

TABLE 1

Computational Complexity Analysis: Channel Estimation Schemes with Block-Type Pilot Arrangement

Estimation Scheme	Complexity	Comments
LS Estimator	Low	Simple vector division.
OLR-MMSE Estimator	Moderate	Avoid matrix inversion and also simplify the matrix operations to the calculations between a low-rank diagonal matrix and a unitary matrix.
MMSE Estimator	High	Matrix inversion and other operations with size N, where N is the DFT size (typically 256, 512, 1024, or 2048).

TABLE 2

Computational Complexity Analysis: Channel Estimation Schemes with comb-Type Pilot Arrangement

Estimation Scheme		Complexity	Comments
LS Estimator with 1D Interpolation	LI	Lowest	Simple Estimation and Interpolation Methods
	SOI	Low	
	SCI	Moderate	Interpolation methods are relatively complex with fitted polynomial search, Low pass convolution and DFT/IDFT calculations
	LPI		
	TDI		
ML Estimator		High	Matrix inversion with size (L+ 1), where L ranges from N/32 to N/4 and other matrix operations with size N.
PCMB Estimator		High	Tracking the number of resolvable paths(M) and channel delays, and matrix inversion with size M, and other matrix operations with size N.

VI. CONCLUSION

In OFDM systems, efficient channel estimation schemes are essential for coherent detection of a received signal. After multi-carrier demodulation, the received signal is typically correlated in two dimensions, in time and frequency. By periodically inserting pilots in the time-frequency grid to satisfy the 2D sampling theorem, the channel response can be reconstructed by exploiting its correlation in time and frequency. This paper fully reviews channel estimation strategies in OFDM systems. It describes block-type pilot-channel estimators, which may be based on least square (LS), minimum mean-square error (MMSE) or optimal low-rank MMSE (OLR-MMSE), with or without a decision feedback equalizer. It also analyzes the comb-type pilot channel estimators, which can be an LS estimator with certain 1D interpolation, Other channel estimators are introduced, such as the estimators based on 2D pilot arrangement with simplified 2D interpolation, the iterative estimators based on iterative filtering and decoding, and the estimators for the OFDM systems with multiple transmit antennas.

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