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Critical Analysis of Statistical Foundation of Irreversible Thermodynamics Subject To Coarse Grain Entropy

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***Abstract*-The extensive study of statistical foundation of irreversible thermodynamics has been carried out. The critical analysis reveals that the conceptual problems are the probabilities in statistical mechanical explanations of thermodynamic phenomena, the issue of irreversibility and the associated thermodynamic arrow of time. The microdynamical origin of thermodynamic irreversibility represents perhaps the most important unsolved problem in the foundations of statistical mechanics. The coarse-graining approach to statistical mechanics and the problems it faces in accounting for thermodynamic irreversibility**

Index Terms -statistical mechanics, irreversibility

I. Introduction

One of the basic aims of the theory of statistical mechanics is to emphasis on a theory formulated in terms of the dynamical laws governing the motion of the microscopic constituents of a thermodynamic system. Although the enormous progress has been made on both the physical and mathematical approaches of statistical mechanics, but the problem remains largely unresolved to this day. The main conceptual problems are the probabilities in statistical mechanical explanations of thermodynamic phenomena, the issue of irreversibility and the associated thermodynamic arrow of time. The microdynamical origin of thermodynamic irreversibility represents perhaps the most important unsolved problem in the foundations of statistical mechanics. The coarse-graining approach to statistical mechanics and the problems it faces in accounting for thermodynamic irreversibility

II. Critical Analysis

The first objection is based on the observation that the coarse-graining approach chooses to equate 'true equilibrium states' on the one hand with 'quasi-equilibrium states' on the other. This coarse-grainer justifies on the grounds that there are no observable differences between the two types of states. However, this appeal to appearances fails when attention is paid not only to momentary differences between the two types of states, but also to differences over time in the behaviour generated by these two types of states. Other is the procedure of coarse-graining introduces an unacceptable element of subjectivity into what should be an objective description of a phenomenon.

The thermodynamic notion of equilibrium is defined in terms of the invariance of the macroscopically observable variables that define the thermodynamic state of the system. In the Gibbs ensemble approach, this notion of time invariance is applied to ensemble distributions rather than individual systems. The equilibrium distribution which remains invariant under the dynamical evolution:

$$\frac{\partial \rho_0}{\partial t} = 0 \tag{1.1}$$

This dynamical requirement does not define a unique equilibrium distribution; in fact, infinitely many distributions can be shown to fulfill this requirement. To single out a unique time invariant distribution, the postulate of equal a priori probability is introduced into the formalism. The requirement of time invariance, together with the postulate of equal a priori probability, defines a unique equilibrium distribution, known as the microcanonical ensemble distribution. When equilibrium state evolves in nonequilibrium state, then the distribution function designed for equilibrium state cannot work for nonequilibrium one. For if the member systems of an ensemble obey the Hamiltonian laws of motion, then the time evolution of the ensemble distribution is subject to Liouville's theorem, which can be expressed as follows:

$$\frac{\partial \rho}{\partial t} + \sum_{n=1}^{N} \left\{ \frac{\partial \rho}{\partial q_i} \frac{\mathrm{H}}{p_i} - \frac{\partial \rho}{\partial p_i} \frac{\mathrm{H}}{q_i} \right\} = 0 \qquad (1.2)$$

Conclusion seems unavoidable that the Gibbs ensemble approach is unable to provide a microdynamical account of the evolution of non-equilibrium states into equilibrium states. In response to these problems, Gibbs introduced the so-called coarse-graining method. This is the relevant statistical mechanical quantity for describing the behaviour of a thermodynamic system is not the ensemble density (also called fine-grained density) ρ , but a coarse-grained density $\overline{\rho}$ which takes the same value on microstates.

This idea is implemented by dividing the accessible region of phase space Γ into cells of small but finite volume ω : The value of ω although small as compared to the macroscopic dimensions, must be at least of the order of the measurement resolution, so that all microstates that measurements cannot discriminate between lie within the same coarse-grained cell. The coarse-grained ensemble density is then obtained by taking the average of the ensemble density in each cell:

$$\overline{\rho}(\Omega, q, p, t) = \frac{1}{\omega} \int_{\Omega} \rho(q', p', t)\, d\Gamma \qquad (1.3)$$

As a result, it is possible in principle for the coarse-grained density to evolve, upon the lifting of a macroscopic constraint, in such a manner as to eventually cover the available region of phase space uniformly in a coarse-grained sense. This state of affairs, in which an equal proportion of each cell of the partition on Γ -space is occupied by the ensemble density, will be referred to as the 'coarse grained equilibrium state'. This is the Gibbs official proposal for when equilibrium is achieved.

Before we proceed it is important to notice that the procedure of coarse-graining is not by itself sufficient to guarantee the approach to a coarse-grained equilibrium by an arbitrary coarse-grained density distribution, as can be seen from the following considerations. The representative points of the systems in the ensemble evolve in phase space so as to obey Liouville's theorem at all times, which means that the total volume occupied by the ensemble density remains constant.

In order for the coarse graining strategy to be successful, the dynamical laws governing the time evolution of the microstates must be such that in the course of the time evolution the representative points become widely separated, as different trajectories reach out to different parts of the available region of phase space.

A coarse-grained equilibrium state can then be obtained as a result of the representative points becoming progressively more spread out over all of the available phase space, eventually distributing them such that all coarse-grained cells are occupied by equally many representative points. For this to be the case, the dynamical evolution of the system has to satisfy the mixing condition or any of the stronger conditions in the ergodic hierarchy. In particular, in the case of a mixing system, a coarse-grained equilibrium state will be obtained in the infinite time limit. But the fact that we need such mixing condition ns shows that the coarse-graining procedure is not by itself sufficient to achieve an approach to a coarse-grained equilibrium state.

Gibbs resorted by replacing the fine-grained ensemble distribution with a coarse-grained distribution. Instead of demanding a fine-grained approach to equilibrium, the aim is now to demonstrate that arbitrary distributions will evolve to equilibrium in the coarse grained sense. The viability of this strategy is premised on the fact that the time evolution of the coarse-grained distribution is not constrained by Liouville's theorem.

Despite its obvious attractions, the coarse-graining method has provoked many criticisms, among which the assertion that the method of coarse-graining introduces an element of subjectivity into statistical mechanics features most prominently. This criticism derives from the fact that, since physics itself cannot generate a rule which determines the size of the coarse-grained cells; the choice of a specific coarse-graining is made purely on the basis of the actual value of the observer's measurement resolution.

Ensemble distribution does indeed depend on the observer's measurement resolution. This dependence arises from the fact that the higher the measurement resolution, the smaller the cells of the coarse-graining has to be chosen in order for the impression of uniformity to be created for the observer. It is this observation that has led to the criticism that the coarse-graining approach is essentially a subjective approach to statistical mechanics.

Gibbs ensemble approach groups together all microstates compatible with the macroscopic state. In trying to get a grip on the problem of the approach to equilibrium faced by the ensemble approach, the coarse-graining approach introduces a subdivision of the microstates compatible with the macrostate M; grouping together those microstates which, although different, cannot be distinguished from each other by measurements of a given resolution.

Microstates are grouped together in coarse-grained cells on the basis of their indistinguishability with respect to quantities that do not figure in the thermodynamic description of the system, and that therefore add a layer of detail to the system's description that goes beyond the purely thermodynamic point of view.

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