

Study of Orbital Angular Momentum in Electromagnetic waves using UCA

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Abstract—Concept of Orbital Angular momentum with the help of mathematical equations has been discussed. An investigation of 4, 6 and 8 elements uniform circular array (UCA) for different Orbital Angular Momentum (OAM) modes is carried out. The effect of varying the uniform circular radius is also discussed. The application of OAM in microwave frequency to enhancing the channel capacity is discussed. Uniform circular antenna array for all possible combination has been simulated and its results are demonstrated for different physical and electrical characteristic

Keywords— Orbital Angular Momentum (OAM), Spin Angular Momentum (SAM), Uniform Circular Array (UCA), MIMO.

I. INTRODUCTION

According to Ericson Mobility Report June 2017, the number of worldwide mobile Subscriptions will go from 7.5 billion (year-2016) to 9.2 billion (year-2022), and worldwide data traffic per smartphone will go from 2.1GB/Month (year-2016) to 12GB/Month (year-2022). Wireless data rate requirement is increasing exponentially day by day and fulfilling this requirement is a great challenge before the researchers, because of the limited radio spectrum and polarization availability. To meet these challenges, high density adopting coding and channel sharing techniques has been developed to increase channel capacity and efficiency. By using these techniques optimum channel capacity and spectrum efficiency has been achieved but the requirement is still more. Recently a concept of Orbital Angular Momentum (OAM) which is well studied in optical and quantum physics has been used in microwave frequency to increase the channel capacity [1-3]. In microwave communication there are two type of momentum one is Spin Angular Momentum (SAM) or polarization which is related to the rotation of electric field vector. Other is OAM which defines the twisting of wave front. Degree of twisting defines the mode number of OAM wave. Each OAM mode can be used as independent channel. And by multiplexing of all these modes we can increase channel capacity. There are many ways to generate OAM wave in radio communication, for planar and low profile antenna, Phased Uniform Circular Array (UCA) is a simplest way to produce $\pm l$ th order OAM mode with N antenna element circular array [4].

II. MATHEMATICAL ANALYSIS OF OAM WAVE

OAM mode l of an N element antenna array is define by the relative electrical phase difference θ between each successive antenna element which is define as $\theta = 2\pi l/N$ [5-6]. The largest OAM mode number (l_{max}) by N element antenna array is $-N/2 < l_{max} < N/2$ [6]. The electrical field vector $E(r)$ at a detection point $P(r, \theta, \phi)$ can be given by.

$$\begin{aligned} E(r) &= -j \frac{\mu_0 \omega}{4\pi} \sum_{n=0}^{N-1} e^{il\varphi_n} \int \frac{e^{-ik|r-r'_n|}}{|r-r'_n|} dV'_n \\ &\approx -j \frac{\mu_0 \omega d}{4\pi} \frac{e^{-ikr}}{r} \sum_{n=0}^{N-1} e^{-i(kr_n - l\varphi_n)} \\ &\approx -j \frac{\mu_0 \omega d}{4\pi} \frac{e^{-ikr}}{r} N i^{-l} e^{il\varphi} J_l(ka \sin\theta) \end{aligned}$$

Where j is the constant current density vector of the dipole, d is the electric dipole length, μ_0 is the magnetic permeability in the vacuum, and ω and k are the angular frequency and wave vector, respectively. $J_l(ka \sin\theta)$ is the Bessel function of the first kind and first order, $\varphi_n = 2\pi n/N$, where $n = 0, 1, 2, \dots, N-1$, is the angle of the array element position, and " a " is the radius of the array. Here keeping the phase constant stationary to obtain the behavior of the phase i.e. $(k.r + l.\varphi) = \text{constant}$. Phase fronts of beams with different orbital angular momentum, are shown in Fig. 1.

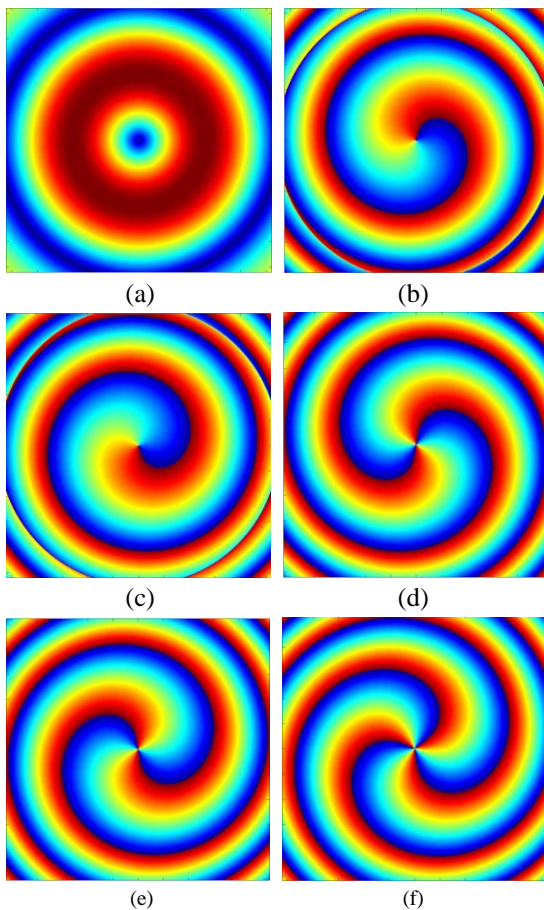


Fig 1 Phase fronts for different orbital angular momentum mode.
(a) $l = 0$, (b) $l = -1$, (c) $l = +1$, (d) $l = -2$, (e) $l = +2$, (f) $l = +3$.

III. SIMULATED ANTENNA ARRAY AND ITS RESULTS

A. Single antenna element for array

A single inset feed patch antenna has been simulated on FR-4 substrate at 10 GHz, which is shown in Fig 2(a) and its dimensions are given in table 1. This antenna has been used for N element array.

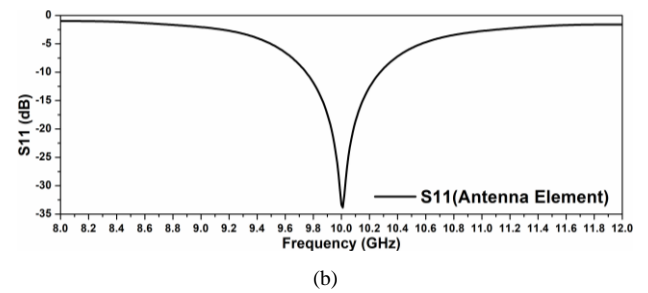
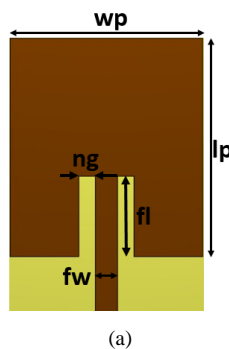


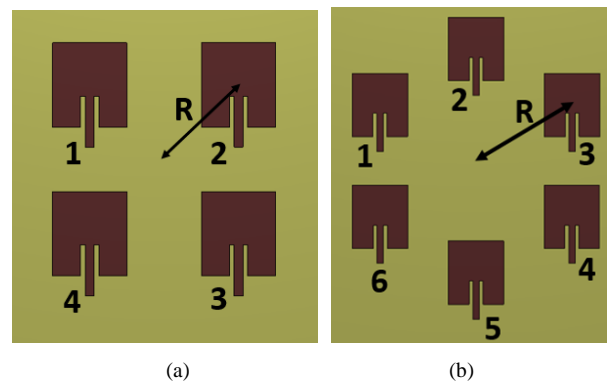
Fig 2 (a) Single patch antenna element, (b) S_{11} of single patch antenna

TABLE I. ANTENNA ELEMENTS PARAMETERS

Variable	Name	Value(mm)
wp	Width of patch	6.2
lp	length of patch	6.7
fl	Inset feed length	3.0
fw	Inset feed width	0.7
ng	Notch gap	0.2

B. Circular Array for different Varying Parameters

Designed patch antenna is used as an antenna element in $N = 4, 6$ and 8 antenna element array as shown on Fig 3. As discussed before OAM mode $\pm l$ depends on the successive electrical phase difference between each antenna elements. For $N = 4, 6$, and 8 the electrical phase difference between successive antenna element for $l = +1$ mode will be $\theta = 90^\circ, 60^\circ$, and 45° respectively. To create electrical phase difference a phase divider network has to be made by creating the physical path difference. If we take the example of Fig 3(c) which is an eight element antenna array, creating the phase divider network for $l = -1$ will become very complex as the progressive phase difference for element five with respect to element one is 180° same thing is applied to the element 6, 7, and 8 with respect to 2, 3 and 4 respectively. And also as length of transmission line increase the decrease in relative amplitude difference will become more dominant.



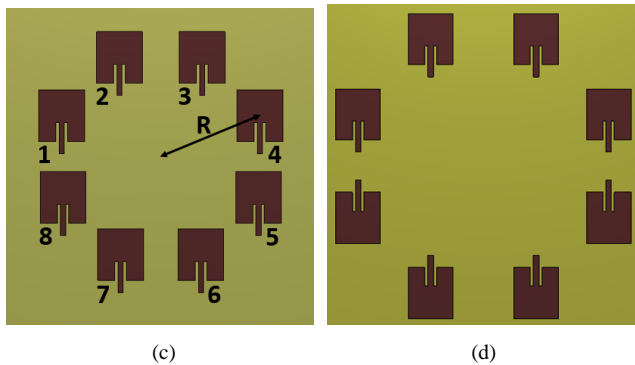


Fig 3 Prototype Antenna Array for (a) 4 Element (b) 6 Element (c) 8 Element (d) 8 Element by rotating the lower half elements by 180° .

To remove the complexity of designing of phase divider network the lower half elements can be rotate on their own axis by 180° as shown in Fig 3(d), and by doing so there is no need to create a line which can provide 180° phase deference. The line design for element one can be use for element five also. Similarly line connecting to element 2, 3 and 4 similar line can be use to connect element 6, 7 and 8 respectively. Fig 4 shows the relative electrical phase difference created by phase divider network. The relative phase difference at 10 GHz is 45° .

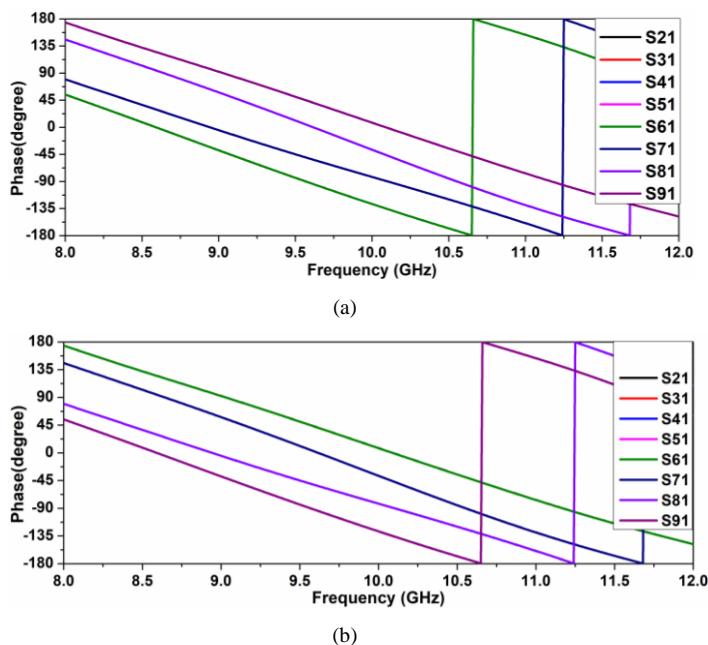


Fig 4 Simulated Successive phase difference between antenna elements for (a) $l = -1$ and (b) $l = +1$

Directivity plot for OAM mode 0, +1, +2 and +3 is shown in Fig 5(a). For mode 0, there is no vortex in boresight direction which is one of the important characteristic of OAM wave so

one can say that zero mode OAM is actually not an OAM wave it is a TEM wave. But in mode ± 1 and ± 2 there is vortex in boresight direction which is because electric fields get subtracted as phase difference increase.

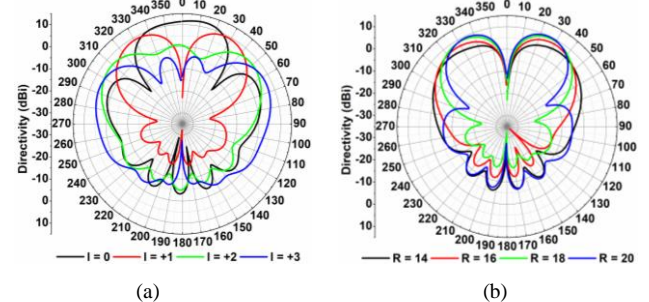


Fig 5 Directivity plot for (a) $l = 0, +1, +2, +3$ for $N = 8$ and $R = 18$ mm (b) Directivity plot for $R = 14, 16, 18, 20$ for $N = 8$ and $l = +1$.

Fig 5(b) shows the effect of varying radius R which is $R = 14, 16, 18$ and 20 mm, by keeping $N = 8$ and $l = +1$. It shows that the flaring of the OAM wave or Vortex solid angle get decrease by increasing the Array radius. Fig 6(a) shows the directivity plot for $l = -1, -2$ and -3 by taking $R = 18$ and $N = 8$. Directivity pattern of $\pm 1, \pm 2$ and ± 3 modes are identical but the phase pattern is different, for $+1$ mode twisting of phase front is clockwise and for -1 anticlockwise as shown in Fig 7. Same directivity plot is also plotted for $N = 6$ and 8 which shows the same characteristic but these plots has not been shown here for the shake of brevity. Fig 6(b) shows the effect on OAM directivity pattern by increasing the number of antenna element N . By increasing N the purity of directivity pattern of higher mode increase. The effect is more visible for OAM mode $l = -3$, because to increase mode number, number of antenna element has to increase to improve the purity of OAM phase pattern.

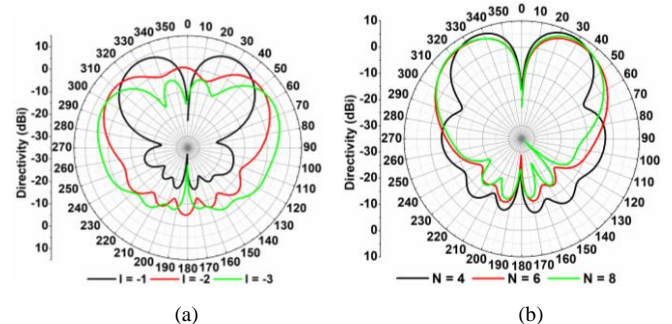


Fig. 6 Directivity plot for (a) $l = -1, -2, -3$ for $N = 8$ and $R = 18$ mm. (b) Directivity plot for $N = 4, 6, 8$ for $l = +1$ and $R = 16$ mm.

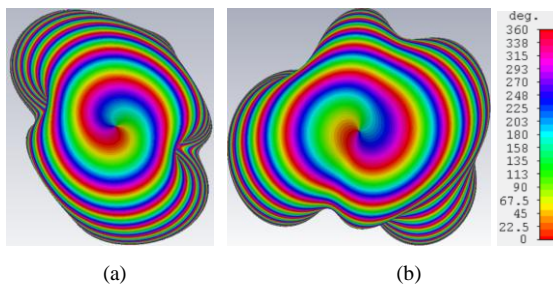


Fig. 7 (a) Phase plot for $l = +1$ and (b) Phase plot for $l = -1$ for $N = 8$.

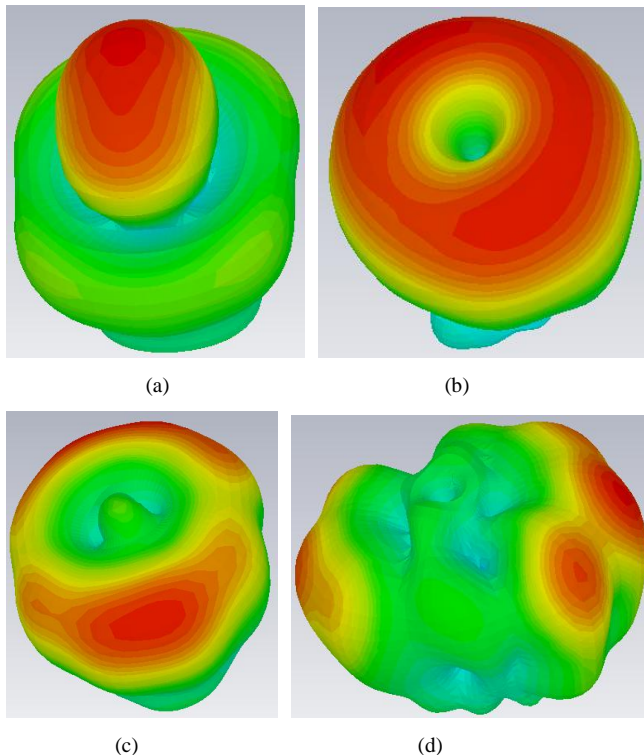


Fig. 8 Simulated four-field radiation pattern of $N = 8$, $R = 18$ mm (a) $l = 0$ (b) $l = -1$ (c) $l = -2$ (d) $l = -3$

IV. CONCLUSION

The effect of array radius on OAM radiation pattern for different modes and for different number of element has been demonstrated. Simulated phase pattern for OAM mode +1 and -1 is demonstrated. Simulated phase pattern and mathematically calculated phase pattern is compared. This results would be used for implementation of reconfigurability in OAM antenna array for MIMO application.

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