Fractional Order Modelling for Different Temperature Processes

Ashna Virdikar, Devarshi Upadhyay, Ashutosh Yadav, and Dr. Saurav Mitra

Electronics Engineering Department, Vidyalankar Institute of Technology, Mumbai, India

Abstract-In this paper, a new approach is proposed to model the dynamics of temperature profile and temperature tracking of three different processes by using fractional calculus. Model analysis is exhibited by comparing fractional ordered PID controllers which are also tuned by using fractional calculus, and traditional integer ordered PID controllers. It is seen that by using fractional order tuning methods, more accurate results are obtained. Main purpose of this review paper is to show the benefits of the fractional calculus based modelling in system temperature profiling toward better results. Comparing the results of fractional order PID and traditional PI/PID controllers it is clear that the proposed tuning method is more beneficial for better temperature profiling.

Keywords- Temperature control, temperature, fractional order controllers, mathematical model, fractional calculus.

I. INTRODUCTION

Temperature modelling is an integral part of most of the industrial processes where this main process parameter is generally controlled by using traditional PID based controllers of integer order. In modern control theory of dynamic processes, applications are now found for fractional order based control for the systems which are containing derivatives of non-integer order [1-2]. Such systems with fractional dynamics are better described by derivatives and integrals of fractional orders which describes non-locality of power-law and also long range dependence of fractal properties [1-5]. To understand the anomalous behaviour of different dynamic systems used in process industries, in space crafts and in other chaotic systems, fractional order modelling are found to have a better fit than integer order system.

Main objective of this paper is to show a new tuning method of controller design by using fractional order differentiation and integration for spatially distributed temperature profiling and tracking. This new scheme of fractional order integral and derivative controller tracks the spatial temperature with more accuracy [3]. It helps to regulate temperatures at given sensor locations better which in turn ensures that the modelled temperatures are very close to the monitored parametric values. This paper reviews fractional order model for three different temperature processes in highly dynamic environments. In temperature control, there are mainly three important tasks. These are temperature set point regulation, temperature profile tracking, and temperature uniformity control [3-4]. In batch process applications like chemical reactor operation, and precision heat treatment for materials, first the system temperature is required to be raised to a prescribed level. This is ensured by temperature profile tracking and set point regulation. By temperature uniformity control, a spatially distributed uniform temperature profile is achieved within predefined locality [3].

The three modes of PID controller act differently to produce the overall control signal. Each of these three modes has their own limitations. When the set-point goes through a large dynamic change, due to the reset windup error in integral controllers, the integral error build up to a significant level. This leads to overshoot and the error continues to rise [3]. It happens because the integral grain change is comparatively a slower process and the generated control signal is directly proportional to the cumulative error produced over time. For a system step response, this integral grain forces the steady state error towards zero. The proportional gain on the other hand, controls the set-point of the controller [3]. If the temperature is below the set-point, then the output will be kept on for longer time. On the contrary, if the temperature is above the set-point, the output will be kept off. The derivative gain produces control signal which is directly proportional to the rate of change of output error. As this most is fast, at certain times, it produces system instability [3-5].

The three fractional order based temperature models mentioned in this paper are Quanser heat flow equipment (HFE) simulation [3], metallurgical annealing furnace tunnel heat treatment [4], and temperature distribution within a spinning satellite [5]. Detailed analysis related to the process model development using fractional calculus may be found in references [1-5]. Our contribution in this paper is to show through accumulated results from three papers, that fractional calculus based modelling strategy produces better modelling accuracy, but at the cost of rigorous mathematical model development and analysis.

II. LITERATURE SURVEY

The idea of fractional order calculus is more than 323 years old and was initiated by French mathematician Guillaume de l'Hôpital in his letter to German mathematician Leibniz. The other eminent contributors in this field are Euler (1730), Lagrange (1772), Laplace (1812), Fourier (1822), Abel (1823), Liouville (1832), and Reinmann (1876) etc. From the interpretations obtained by so many contributors over the years, now the geometrical interpretation or physical meaning of fractional order calculus exists, but it is not as straight forward as the integer order derivatives.

After an extensive literature survey in the field of fractional calculus based process modelling, it is found that the pioneering research work in this field was carried out by Manabe (1960-61), Oustaloup (1981), and Axtell (1990) for temperature related systems. It is also found that there are many applications of fuzzy-neural network based PID controller to control temperature based engineering systems. Galan *et. al.* showed that for temperature tracking of injection moulding, fuzzy logic is necessary to enhance the performance of PID controller. Also an application was mentioned in the paper of Dihac *et. al.* that uses PID controller for rapid thermal process control (1992). A neural fuzzy inference network for temperature control of a water bath system was proposed and compared with the performance of PID based control by Lin *et. al.* (1999). Ramos *et. al.* used PID controller for controlling temperature of hot bath (2005). Juang and Chen (2003) had put forth the idea of a TSK-type recurring neural fuzzy network which was based on the direct inverse control configuration. This configuration does not require any prior knowledge of the plant order. So far, fractional order calculus had been studied only as an alternative to calculus in a mathematical domain Debanatho (2004). Chen (2004) had predicted that fractional order calculus will have various applications in the field of mechatronics and biological systems. Literature of a fractional order based PID control model which was proposed by Ahn *et. al.* has also been reviewed (2008).

III. BASICS OF FRACTIONAL CALCULUS, & THERMAL MODELLING WITH PID

Some basic formulae for the fractional order calculus are presented in this section. These will be applied in subsequent sections. Fractional order integral is defined as:

$$I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{(\alpha-1)} f(\tau) d\tau, \quad (1)$$

where $t > 0, \alpha \in R^+$. For the fractional derivative, Caputo derivative is used. Caputo derivative is defined as:

$$D^{\alpha} f(t) = \frac{d^{\alpha} f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(\alpha - n)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, \quad (2)$$

where $(n-1) < \alpha < n$. The Euler's Gamma function is given as :

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt, \quad \text{for } x > 0. \quad (3)$$

With the special case where x = n,

$$\Gamma(n) = (n-1)(n-2)\ldots(2)(1) = (n-1)!$$

Considering all the initial conditions to be zero and using Laplace transformation, the fractional PID controller equation as given by Podlubny is (1999)

$$C(s) = K_p + K_i s^{-\mu} + K_d s^{\lambda} \quad (4)$$

If taken $\mu = \lambda = 1$, we obtain the classic PID controller. With $\mu = 0$, it is the PD controller and if $\lambda = 0$, it is a PI controller. Also we can consider a fractional order $I^{\alpha} D^{\beta}$ controller which is a special case of fractional PID controller with $K_p = 0$.

In papers of Oustaloup *et. al.* (1995) and Podlubny *et. al.* (1997) the idea of using fractional order controllers for temperature control was proposed for the first time. Monje *et. al.* (2005) claimed the following advantages of fractional order PID (FOPID) over traditional PID controller

1. No steady state error
2. Better phase margin & gain crossover frequency specifications
3. Better gain margin & phase crossover frequency specifications
4. Robustness to variations in the gain of the plant
5. Robustness to high frequency noise
6. Good output disturbance rejection – as FOPID controller has five tuning parameters, five specifications can be met by the closed-loop system. The five tuning parameters are K_p, K_i, K_d, α and λ.

$I^{\alpha} D^{\beta}$ Controller Tuning - The main problem that arises in designing an appropriate controller for temperature process control is in variation of its transfer function. The transfer function varies with perturbations and is highly dependent on changes in other temperature related parameters as well. The transfer function for output temperature changes with the accumulated error over time. It happens so because the temperature changes according to the power input to the system. In Petras and Vinagre (2002), it is shown that the transient unit response of a heat solid system can be better modelled by fractional order calculus. In 2005, Aoki *et. al.* reported that the heat transfer coefficients can also be of fractional order. Thus, by using a fractional model we can approximate the time dependent behaviour of systems better compared to its integer order model. As the thermal diffusion process is involved with process of heat transfer, a half order controller matches the physics

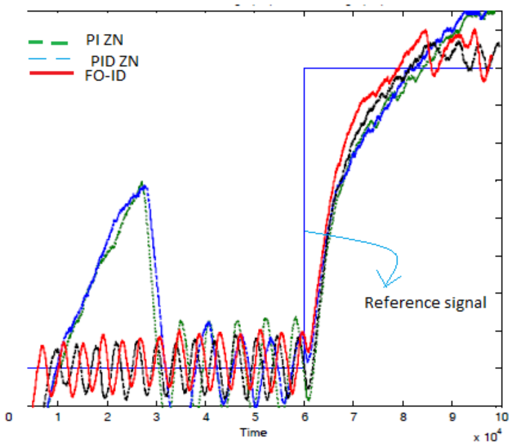

Fig.1. Comparison of output graphs between PI, PID, FO-ID, FO-PI controllers with the reference signal [3]

of the system better. Therefore in this paper, it is proposed that the idea of using a $I^{\alpha} D^{\beta}$ controller which gives more accurate temperature tracking compared to a traditional integer order controller. The FOPID, PI and FO-ID controller output graphs are shown in Figure 1. The graphs are compared to observe for more stable controller output response. They are compared with a common reference signal which is also shown in the Figure 1. It is clearly seen that the fractional order controllers perform far better than the integer ordered PI/PID controllers. The performance of FOPID is better because FOPID tuning methods are better than that of traditional PID controller.

IV. ANNEALING PROCESS MODEL WITH PID/FOPID & SIMULATION RESULTS

The technical event of furnace heating can be mathematically modelled by using the ordinary differential equations. Here, the heating process of annealing furnace is considered as a very relevant example in the field of thermal engineering. In this process, first the steel pipes in the annealing furnace are heated to a predefined temperature so that the temperature within the pipe volume becomes homogenous. The temperature within the pipe is sustained at this raised level for a certain period in the consequent step. Finally, in the third step, the pipe is cooled down slowly and its temperature is brought back to normal environment condition. In Figure 2, the longitudinal section of the annealing furnace is shown. The section is found to contain the centres of the external wheels. The heating resistance passes through the entire surface of the furnace walls. The thermal input to the furnace and the output of the furnace are shown in Figure 2. The rollers are used to move the pipe from the input door to the heating position. Post heating process, they are taken out of the system using the output door.

Model of the Heating Process – The final thermal structure is obtained from the energy balance equation. The structure is shown in Figure 3. Here

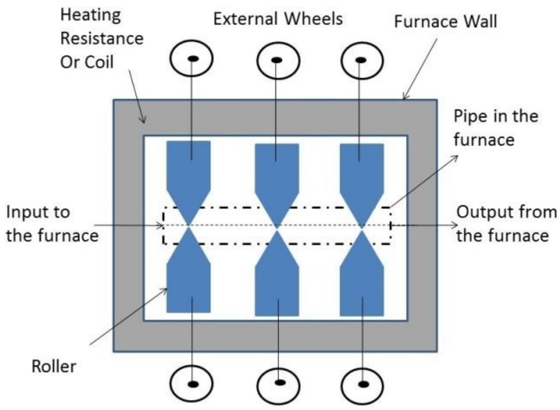

Fig. 2.The longitudinal section of the furnace which contains the centres of the external wheels [4]

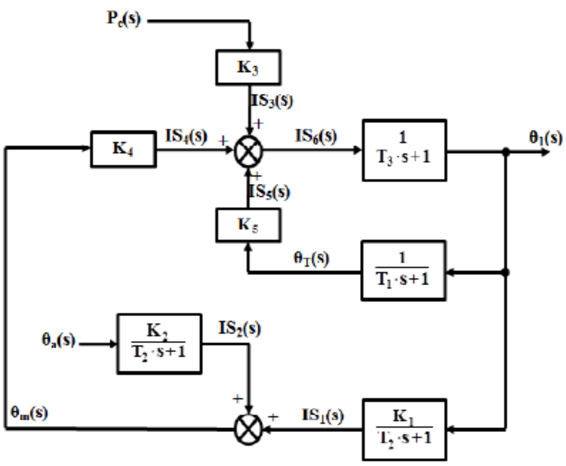

Fig. 3. Final Laplace domain structure of the thermal annealing process obtained through energy balance equation [4]

$\theta_1(s)$ is the temperature of the air from the furnace; $\theta_m(s)$ is the temperature of the furnace walls; $P_e(s)$ is the is electrical power generated by the heating source; and $\theta_a(s)$ is the ambient temperature.

Proposed Temperature Control Scheme – The temperature control is obtained through fractional order based PID control scheme of the given form:

$$H_{FOPID}(s) = \frac{Kc\left(1 + \frac{1}{T_I s} + T_d s^{\mu}\right)}{T_f(s+1)} \quad (5)$$

When the value of μ = 1, we get integer order PID controller of the following form:

$$H_{PID}(s) = \frac{Kc\left(1 + \frac{1}{T_I s} + T_d s\right)}{T_f(s+1)} \quad (6)$$

The necessity of using a fractional order controller occurs due to the fact that the overshoot generated while using integer order PID controller has to be improved. Higher value of overshoot generated by integer order PID causes more instability in the system.

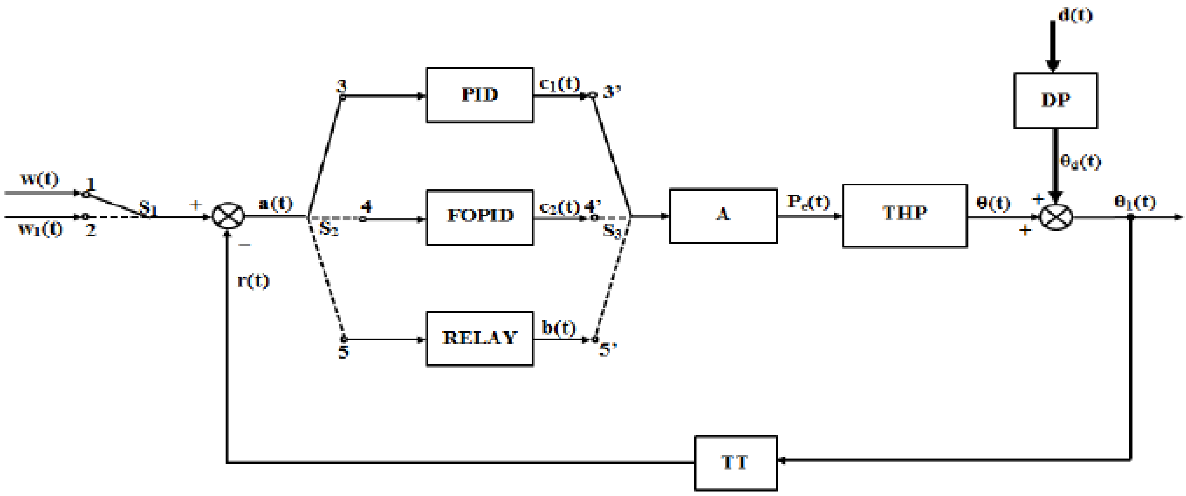

Fig. 4. Proposed temperature control structure for thermal (annealing) process using integer and fractional order PID schemes [4]

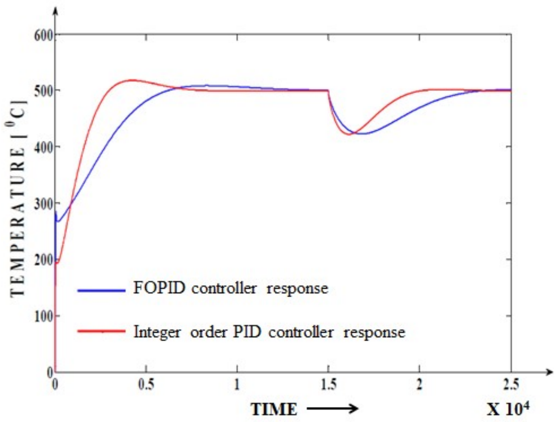

Fig. 5. The comparative graph between the control system step responses, for integer order PID and FOPID controllers [4]

In Figure 5, the comparative graph between the control system step responses is presented (for the time evolutions of $\theta_1(t)$ signals), while using simple integer order PID and FOPID controllers. The temperature set point condition for both cases is set to 500 $^\circ C$ (over the initial conditions of 20 $^\circ C$). Comparing the graphs in Figure 4, it is found that in both cases, the steady state values of the two responses are equal with the imposed temperature value of 500 $^\circ C$ (in both cases the steady state error has value 0 $^\circ C$). In integer order PID controller, the obtained overshoot σ_1 = 3.52%, whereas in FOPID controller, the obtained overshoot σ_2 = 1.7% which is lower than the imposed limit of 2%. The settling time obtained in case of simple PID controller t_{s2} = 6700s which is smaller than the settling time obtained in the case of FOPID controller t_{s3} = 10200s. The following process was for modelling and controlling the heating (in the thermal annealing process) associated to a tunnel furnace used in metallurgical industries. In order to achieve improved performance in overshoot, FOPID gives better performance with higher system stability.

V. SPINNING SATELLITE THERMAL DISTRIBUTION - FRACTIONAL MODELLING

In the design of artificial satellites, it is important to determine the temperature distribution on the surface of the spacecraft. An interesting and special case was reported by Machado and Galhano about the temperature fluctuations in the skin of the satellite due to spinning of the vehicle [6]. Lee *et. al.* studied and analysed the unsteady-state temperature distribution of micro-satellite under stabilization effects [7]. Liu *et. al.* gave an improved solution to thermal network problem in heat transfer analysis of spacecraft [8]. They reported that the temperature distribution at satellite surface is affected by solar absorptivity. Numerical simulation on antenna temperature field of complex structure satellite in solar simulator can be found in research paper by Liu *et. al.*[8]. Yang *et. al.* studied thermal analysis for folded solar array of spacecraft in orbit [9].

Spinning satellite is a satellite that spins in a fixed axis by using gyroscopic effect. Entire satellite stably spins around its own axis as a whole and acts as a gyroscope [5]. Spinning of satellite can be achieved using rods attached to the satellite with coils around the rods. Current passing through the rods helps to generate electromagnetic field around the rods. This self generated magnetic field interacts with Earth's magnetic field and the rods begin to spin [5]. If the craft is thin-walled, then there is no radial dependence. Hrycak showed the approximation of the non dimensional temperature field at the equator of the rotating satellite [10]. It obeys the mathematical equation:

$$\frac{d^2T}{d\eta^2}+b\frac{dT}{d\eta}-C\left(T-\frac{3}{4}\right)=-\frac{\Pi c}{4}\frac{F(\eta)+\frac{\beta}{4}}{1+\frac{\Pi\beta}{4}},$$

(7)

where, $b=\frac{4\Pi^2 r^2 f}{a}$, and

$$C=\frac{16\Pi S}{\gamma T_\infty}\left(1+\frac{\Pi\beta}{4}\right),$$

$$T_\infty=\left(\frac{S}{\Pi\sigma\varepsilon}\right)^{\frac{1}{4}}\left(\frac{1+\frac{\Pi\beta}{4}}{1+\beta}\right)^{\frac{1}{4}}.$$

(8)

Here, in equation (7) and (8), r is the radius of spacecraft, a is the thermal diffusivity of the shell, β is the ratio of the emissivity of the interior shell to the emissivity of the exterior surface, f is the rate of spin, S is the net direct solar heating, ε is the overall emissivity of the exterior surface, γ is the satellite's skin conductance and σ is the Stefan–Boltzmann constant [5]. η is an independent variable and is the longitude along the equator with the effect of rotation subtracted out $(2\Pi\eta=\phi-2\Pi ft)$. T_∞ or the reference temperature equals the temperature that the spacecraft would have if it had spun with infinite angular speed sp that the solar heating would be uniform around the craft. We non dimensionalized the temperature with respect to T_∞ [5]. Using above equations the following graphical representations are shown in [5]. We shall site some of the results from the same paper which will help our explanation.

The graphs in Figure 6 and 7 shows that there is a variation in non-dimensional temperature y(η) which is a function of longitude along the equator with the effect of rotation subtracted out, given by η for spinning rate v_o. For Figure 6, the v_o value is zero as the satellite is assumed not to be spinning. For Figure 7 v_o or satellite spinning rate is assumed to be 0.375 rpm [5]. It can be observed from these two figures that there are higher temperature fluctuations along the equator in longitude, when satellite is in motion along the axis. Thus, it may be concluded that, when the satellite spins with a constant rate there are more temperature fluctuations. The whole process model shows consistent accuracy with fractional calculus based modeling [5].

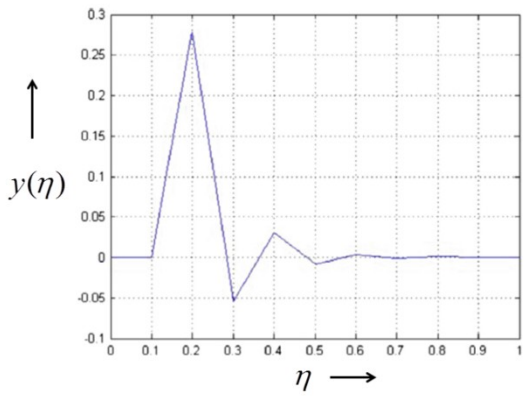

Fig. 6. For μ = 0.75, α = 1.5, observed variations of the non-dimensional temperature is shown, as a function of η and spinning rate v_o. Value of $v_o = 0$ as satellite is not spinning, [5]

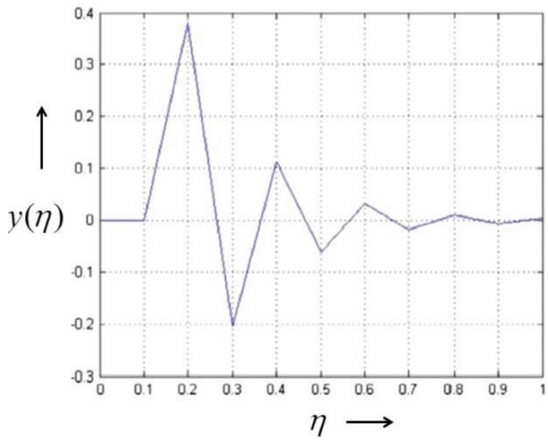

Fig. 7. For μ = 0.75, α = 1.5, observed variations of the non-dimensional temperature is shown, as a function of η and spinning rate v_o. Value of $v_o = 0.375$ rpm, spinning satellite [5]

VI. CONCLUSON

This paper proposes in favour of the effectivity of the fractional calculus based new controller tuning method. The advantages of fractional integral and derivative $I^{\alpha}D\ (¿¿\beta)\ ¿$ type of controller are explained through valid examples. Performance of FO-ID controller is compared to integer ordered PI/PID controller and the accuracy of temperature tracking is measured for both the cases. It is found from the results that the FO-ID controller has more accuracy than integer order simple PID controllers. Also we have come to the conclusion that in case of integer type of integrator, the relative dead time is very small. However, some discrepancies in the simulated results were observed in the experimental results owing to the inaccurate modelling of the temperature system. In order to obtain the imposed performances (especially for less overshoot) for the control system, a fractional-order controller is used. An analytical solution of Caputo fractional differential equation for temperature distribution within a spinning satellite is reported here. The result is given in terms of Wright generalized hyper-geometric function by using fractional calculus approach.

REFERENCES

[1] A. Loverro, “Fractional Calculus: History ,Definitions and Applications for the Engineer,” Rapport technique, University of Notre Dame, Notre Dame, IN 46556,U.S.A, May 2004

[2] Mihailo P. Lazaveric, Milan R. Rapaic and Tomislav B. Sekara,”Introduction to Fractional Calculus with Brief Historical Background,” University of Belgrade, University of Novi Sad,

[3] Hyo-Sung Ahn, Varsha Bhambani and Yangquan Chen, “Fractional-order integral and derivative controller for temperature profile tracking”, Utah State University, UT 84322-4160,USA,Journal of Indian Academy of Sciences, *Sadhana,* Vol.34, Part 5, October 2009, pp.833-850

[4] Vlad Muresan, Mihail Abrudean and Honoriu Valean, Technical University of Cluj-Napoca, Romania, 2016 20th International Conference on System Theory, Control and Computing (ICSTCC), October 13-15

[5] Jyotindra C. Prajapati, Krunal B. Kachhia and Shiv Prasad Kosta , “Fractional Calculus approach to study temperature distribution within a spinning satellite”, *Alexandria Engineering Journal* (2016) 55, 2435-2350, May 2016

[6] S.R. Lee, Z.C. Hong and K.T. Peng, “Analysis the unsteady-state temperature distribution of micro-satellite under stabilization effects,” COSPAR Colloq. Ser. 10 (1999) 324-332

[7] D. Liu, X. Liu and G. Li, “Prediction on effects of absorptivity and emissivity for surface temperature distribution using an improved thermal network model *, Heat Transf-Asian Res*.39 (7) (2010) 539-553

[8] Y.Liu, G.H. Li and L. Jiang, “Numerical simulation on antenna temperature field of complex structure satellite in solar simulator”, *Acta Astronaut.* 65 (7-8) (2009) 1098-1106

[9] W.H. Yang, H.E. Chang and A. Cai, “Thermal analysis for folded solar array of spacecraft in orbit,” *Appl. Therm. Eng*. 24(4) (2004) 595-607