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On Eccentric Connectivity Index of Transformation Graphs

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***Abstract*: The eccentric connectivity index is the sum of the product of eccentricity and degree of every vertex in G. In this paper, we present upper bounds for the total transformation graphs in terms of order, size and the first Zagreb index of the original graph G.**

Keywords: eccentricity connectivity index; first Zagreb index; transformation graph.

AMS Subject Classification: 05C09; 05C92.

I. INTRODUCTION

Throughout this paper we consider only simple graphs. Let G = (V, E) be a graph with |V | = n and |E| = m. The degree of a vertex V ∈ G is denoted by degG(V) and defined as degG(V)= |{u\ uv ∈ E(G)}|. The eccentricity of a vertex V∈ G is the largest distance between v and u for some u ∈ V(G). We follow [5] for unexplained terminology and notation.

Recently, topological indices are playing vital role in QSPR/QSAR studies due to their predicting power. One of the oldest topological index is the first Zagreb index [4] which has been studied extensively by many researchers [6, 7, 10, 12]. It is defined as:

$$M_1(G) = \sum_{i=1}^{n} \deg(v_i)^2 \tag{1}$$

In fact, we can re-write (1) as,

$$M_1(G) = \sum_{uv \in E(G)} \deg(u) + \deg(v) \tag{2}$$

Sharma et.al., [13] have put forward a novel topological index namely, the eccentric- connectivity index ECI(G) of a molecular graph G. Which is defined as follows

$$ECI(G) = \sum_{i=1}^{n} (e(v_i)\deg(v_i)) \tag{3}$$

For more details on topological indices refer [3, 8, 9, 14, 15].

Let G be a graph. The total graph, usually denoted by T(G) of G has V(G)∪E(G) as its vertex set and two vertices of F (G) are adjacent if and only if they are adjacent or incident in G. Inspired by total graph Wu and Meng [16] have generalized the total graph by defining the following transformation graphs:

Let G = (V, E) be a graph and x, y, z be three variables taking values + or −. The total transformation graph Gxyz is a graph having V(G)∪E(G) as a vertex set, and for α, β ∈ V (G)∪E(G), α and β are adjacent in Gxyz if and only if

1. α, β ∈ V (G), α, β are adjacent in G if x = + and α and β are not adjacent in G if x = −.
2. α, β ∈ E(G) α, β are adjacent in G if y = + and α and β are not adjacent in G if y = −.
3. α ∈ V (G) and β ∈ E(G), α, are incident in G if z = + and α and β are not incident in G if z = −.

Note1. Since there are eight distinct 3-permutations of {+,−}, we obtain eight graphical transformations of G . It is interesting to see that G+++ is exactly the total graph T (G) ofi G and G−−− is the complement of T (G). Also for a given graph G, G++− and G−−+, G+−+ and G−+−, G −++ and G+−− are the other three pairs of complementary graphs. All these transformation graphs are depicted in Figure 1. For basic properties of these transformation can refer [1, 2, 6, 17, 18].

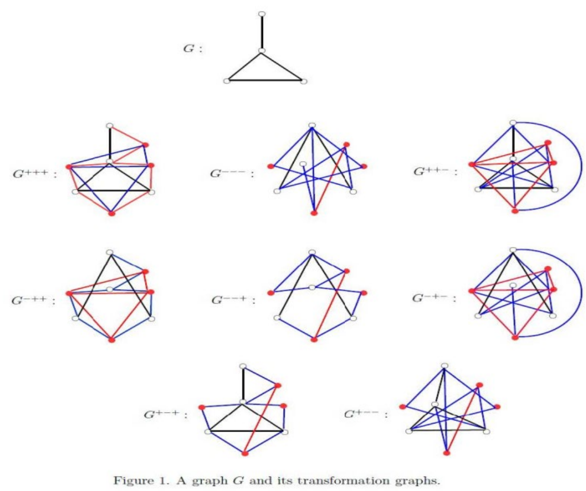

Fig. 1. A graph G and it's trasformation graphs

II. RESULTS

In this section we obtain upper bounds for eccentric-connectivity index of total transformation graphs in terms of order, size and the first Zagreb index.

Theorem1.Let G = (n, m) graph. Then

ECI (G+++) ≤ 2ECI (G) + M1 (G) + 4m + ζ(G) (4) (4)

Where $\xi(G)=\sum_{u_ju_k\in E(G)}^{n} e_G(u_j)(\deg_G(u_j)+\deg_G(u_k))$

Proof. Let G=(V, E) be a graph with V(G) ={v1, v2, v3, …., vn} and E(G)={e1, e2, e3, · · ·.. , em}. Then V (G+++)={v1, v2, v3, · · · , vn, e1, e2, e3, · · · , em}.Clearly |V(G+++)| =m+n. further, diam(G) ≤ diam(G+++) ≤ diam(G)+1.Since, for every v∈V(G) , eG (v) ≤ diam(G).Therefore, for every u ∈ G+++, eG+++(u)≤ eG(u) + 1. Let ui ∈ V(G+++) be the corresponding vertex vi ∈ V (G) and uj ∈ V (G+++) be the corresponding vertex ej∈E(G) in G+++. Then deg(ui)=2deg(vi) and deg(uj)=deg(vi)+deg(vj) where ej= vivj. Therefore

$$ECI(G^{+++})=\sum_{i=1}^{n}(e_{G^{+++}}(u)\deg_{G^{+++}}(u))$$

$$=\sum_{u_i\in V(G^{+++})\cap V(G)}^{n}(e_{G^{+++}}(u_i).\deg_{G^{+++}}(u_i))+\sum_{u_i\in V(G^{+++})\cap E(G)}^{n}(e_{G^{+++}}(u_j).\deg_{G^{+++}}(u_j))$$

Since eG+++(u)≤eG(u)+1 Therefore

$$ECI(G^{+++})\le\sum_{u_i\in V(G)}^{n}[(e_G(u_i)+1).2\deg_G(u_i)]+\sum_{u_ju_k\in E(G)}^{n}[(e_G(u_j)+1).(\deg_G(u_j)+\deg_G(u_k))]$$

$$ECI(G^{+++})=2\sum_{u_i\in V(G)}^{n}(e_G(u_i).\deg_G(u_i)+2\sum_{u_i\in V(G)}^{n}\deg_G(u_i)$$

$$+\sum_{u_ju_k\in E(G)}^{n}(e_G(u_j).(\deg_G(u_j)+\deg_G(u_k))+\sum_{u_ju_k\in E(G)}^{n}(\deg_G(u_j)+\deg_G(u_k))$$

ECI(G+++)≤2ECI(G)+M1(G)+4m+ζ(G),as asserted.

Theorem 2.Let G = (n, m) graph. Then

ECI (G---) ≤ 3(m+n)(m+n-1)-12m-3M1(G) (5)

Proof. Let G=(V, E) be a graph with V(G) ={v1, v2, v3, ……., vn} and E(G)={e1, e2, e3, · · ·.. , em}. Then V (G---)={v1, v2, v3, · · · , vn, e1, e2, e3, · · · , em}.Clearly |V(G---)| =m+n. further, diam(G---) ≤ 3 .Since, for every vЄV(G), eG(v)≤ diam(G).Therefore, for every u ∈ G---, eG---(u)≤ 3.Let ui ∈ V(G---) be the corresponding vertex vi ∈ V (G) and uj ∈ V (G---) be the corresponding vertex ej∈E(G) in G--- .Then degG---(ui)=m+n-1-2deg(vi) and degG---(uj)=m+n-1-(deg(vi)- deg(vj)) where ej = vivj. Therefore

$$ECI(G^{---})=\sum_{i=1}^{n}(e_{G^{---}}(u)\deg_{G^{---}}(u))$$

$$=\sum_{u_i\in V(G^{---})\cap V(G)}^{n}(e_{G^{---}}(u_i).\deg_{G^{---}}(u_i))+\sum_{u_i\in V(G^{---})\cap E(G)}^{n}(e_{G^{---}}(u_j).\deg_{G^{---}}(u_j))$$

Since eG---(u) ≤ 3. Therefore,

$$ECI(G^{---})\le\sum_{u_i\in V(G)}^{n}[3.(m+n-1-2(\deg_G(u_i)))]$$

$$+\sum_{u_ju_k\in E(G)}^{n}[3.(m+n-1-(\deg_G(u_j)+\deg_G(u_k)))]$$

$$ECI(G^{---})=3(m+n-1)n-6\sum_{u_i\in V(G)}^{n}(\deg_G(u_i))$$

$$+3(m+n-1)m-3\sum_{u_ju_k\in E(G)}^{n}(\deg_G(u_j)+\deg_G(u_k))$$

ECI (G---) ≤ 3(m+n)(m+n-1)-12m-3M1(G), as desired.

Theorem3.Let G = (n, m) graph. Then

ECI (G++-) ≤ 4mn+4m(n-4)+4M1(G) (6)

Proof: Let G=(V, E) be a graph with V(G) ={v1, v2, v3, ……., vn} and E(G)={e1, e2, e3, · · ·.. , em}. Then V (G++-)={v1, v2, v3, · · · , vn, e1, e2, e3, · · · , em}.Clearly |V(G++-)| =m+n. further, diam(G++-) ≤ 4 .Since, for every vЄV(G), eG(v)≤ diam(G).Therefore, for every u ∈ G++-, eG++-(u)≤ 4.Let ui ∈ V(G++-) be the corresponding vertex vi ∈ V (G) and uj ∈ V (G++-) be the corresponding vertex ej∈E(G) in G++- .Then degG++-(ui)=m and degG++-(uj)=n-4+(deg(vi)-deg(vj)) where ej = vivj. Therefore

$$ECI(G^{++-})=\sum_{i=1}^{n}(e_{G^{++-}}(u)\deg_{G^{++-}}(u))$$

$$=\sum_{u_i\in V(G^{++-})\cap V(G)}^{n}(e_{G^{++-}}(u_i).\deg_{G^{++-}}(u_i))+\sum_{u_i\in V(G^{++-})\cap E(G)}^{n}(e_{G^{++-}}(u_j).\deg_{G^{++-}}(u_j))$$

Since eG++-(u) ≤ 4. Therefore,

$$ECI(G^{++-})\le\sum_{u_i\in V(G)}^{n}4[m]+\sum_{u_ju_k\in E(G)}^{n}[4.(n-4-(\deg_G(u_j)+\deg_G(u_k)))]$$

ECI (G++-) ≤ 4mn+4m(n-4)+4M1(G), as desired.

Theorem4.Let G = (n, m) graph. Then

ECI(G--+)≤3n(n+1)-6m+3m(m+3)-3M1(G) (7)

Proof: Let G=(V, E) be a graph with V(G) ={v1, v2, v3, ……., vn} and E(G)={e1, e2, e3, · · ·.. , em}. Then V (G--+)={v1, v2, v3, · · · , vn, e1, e2, e3, · · · , em}.Clearly |V(G--+)| =m+n. further, diam(G--+)≤3.Since, for every vЄV(G), eG(v)≤ diam(G).Therefore, for every u ∈ G--+, eG--+(u)≤ 3.Let ui ∈ V(G--+) be the corresponding vertex vi ∈ V (G) and uj ∈ V (G--+) be the corresponding vertex ej∈E(G) in G--+ .Then degG--+(ui)=n+1-deg(vi)- and degG--+(uj)=m+3-(deg(vi)- deg(vj)) where ej = vivj. Therefore

$$ECI(G^{--+})=\sum_{i=1}^{n}(e_{G^{--+}}(u)\deg_{G^{--+}}(u))$$

$$=\sum_{u_i\in V(G^{--+})\cap V(G)}^{n}(e_{G^{--+}}(u_i).\deg_{G^{--+}}(u_i))+\sum_{u_i\in V(G^{--+})\cap E(G)}^{n}(e_{G^{--+}}(u_j).\deg_{G^{--+}}(u_j))$$

Since eG--+(u) ≤ 3. Therefore,

$$ECI(G^{--+})\le\sum_{u_i\in V(G)}^{n}[3.(n+1-(\deg_G(u_i)))]$$

$$+\sum_{u_ju_k\in E(G)}^{n}[3.(m+3-(\deg_G(u_j)+\deg_G(u_k)))]$$

ECI(G--+)≤3n(n+1)-6m+3m(m+3)-3M1(G) , as desired.

Theorem5.Let G = (n, m) graph. Then

ECI(G+-+)≤ 12m+3m(m+3)-3M1(G) (8)

Proof: Let G=(V, E) be a graph with V(G) ={v1, v2, v3, ……., vn} and E(G)={e1, e2, e3, · · ·.. , em}. Then V (G+-+)={v1, v2, v3, · · · , vn, e1, e2, e3, · · · , em}.Clearly |V(G+-+)| =m+n. further, diam(G+-+)≤3. Since, for every vЄV(G), eG(v)≤ diam(G).Therefore, for every u ∈ G+-+, eG+-+(u)≤ 3.Let ui ∈ V(G+-+) be the corresponding vertex vi ∈ V (G) and uj ∈ V (G+-+) be the corresponding vertex ej∈E(G) in G+-+ .Then degG+-+(ui)=2deg(vi)- and degG--+(uj)=m+3-(deg(vi)- deg(vj)) where ej = vivj. Therefore

$$ECI(G^{+-+})=\sum_{i=1}^{n}(e_{G^{+-+}}(u)\deg_{G^{+-+}}(u))$$

$$=\sum_{u_i\in V(G^{+-+})\cap V(G)}^{n}(e_{G^{+-+}}(u_i).\deg_{G^{+-+}}(u_i))+\sum_{u_i\in V(G^{+-+})\cap E(G)}^{n}(e_{G^{+-+}}(u_j).\deg_{G^{+-+}}(u_j))$$

Since eG+-+(u) ≤ 3. Therefore,

$$ECI(G^{+-+}) \le \sum_{u_i \in V(G)}^{n} [3.(2(\deg_G(u_i)))] + \sum_{u_j u_k \in E(G)}^{n} [3.(m+3-(\deg_G(u_j)+\deg_G(u_k)))]$$

ECI(G+-+)≤ 12m+3m(m+3)-3M1(G), as desired.

Theorem6.Let G = (n, m) graph. Then

ECI(G-+-)≤3n(m+n-1)+3m(n-4)-12m+3M1(G) (9)

Proof: Let G=(V, E) be a graph with V(G) ={v1, v2, v3, ……., vn} and E(G)={e1, e2, e3, · · ·.. , em}. Then V (G-+-)={v1, v2, v3, · · · , vn, e1, e2, e3, · · · , em}.Clearly |V(G-+-)| =m+n. further, diam(G-+-)≤3.

Since, for every vЄV(G), eG(v)≤ diam(G).Therefore, for every u ∈ G-+-, eG-+-(u)≤ 3.Let ui ∈ V(G-+-) be the corresponding vertex vi ∈ V (G) and uj ∈ V (G-+-) be the corresponding vertex ej∈E(G) in G-+-.Then degG-+-(ui)=m+n-1-2deg(vi)- and degG--+(uj)=n-4+(deg(vi)-deg(vj)) where ej = vivj. Therefore

$$ECI(G^{-+-}) = \sum_{i=1}^{n} (e_{G^{-+-}}(u) \deg_{G^{-+-}}(u))$$

$$= \sum_{u_i \in V(G^{-+-}) \cap V(G)} (e_{G^{-+-}}(u_i).\deg_{G^{-+-}}(u_i)) + \sum_{u_i \in V(G^{-+-}) \cap E(G)} (e_{G^{-+-}}(u_j).\deg_{G^{-+-}}(u_j))$$

Since eG-+-(u) ≤ 3. Therefore,

$$ECI(G^{-+-}) \le \sum_{u_i \in V(G)}^{n} [3.(m+n-1-2(\deg_G(u_i)))] + \sum_{u_j u_k \in E(G)}^{n} [3.(n-4-(\deg_G(u_j)+\deg_G(u_k)))]$$

ECI(G-+-)≤3n(m+n-1)+3m(n-4)-12m+3M1(G) , as desired.

Theorem 7.Let G = (n, m) graph. Then

ECI(G-++)≤3n(n-1)+3M1(G) (10)

Proof: Let G=(V, E) be a graph with V(G) ={v1, v2, v3, ……., vn} and E(G)={e1, e2, e3, · · ·.. , em}. Then V (G-+-)={v1, v2, v3, · · · , vn, e1, e2, e3, · · · , em}.Clearly |V(G-++)| =m+n. further, diam(G-++)≤3.

Since, for every vЄV(G), eG(v)≤ diam(G).Therefore, for every u ∈ G-+-, eG-++(u)≤ 3.Let ui ∈ V(G-++) be the corresponding vertex vi ∈ V (G) and uj ∈ V (G-+-) be the corresponding vertex ej∈E(G) in G-++.Then degG-++(ui)=n-1 and degG-++(uj)=(deg(vi)-deg(vj)) where ej = vivj. Therefore

$$ECI(G^{-++}) = \sum_{i=1}^{n} (e_{G^{-++}}(u) \deg_{G^{-++}}(u))$$

$$= \sum_{u_i \in V(G^{-++}) \cap V(G)} (e_{G^{-++}}(u_i).\deg_{G^{-++}}(u_i)) + \sum_{u_i \in V(G^{-++}) \cap E(G)} (e_{G^{-++}}(u_j).\deg_{G^{-++}}(u_j))$$

Since eG-++(u) ≤ 3. Therefore,

$$ECI(G^{-++}) \le \sum_{u_i \in V(G)}^{n} [3.(n-1)] + \sum_{u_j u_k \in E(G)}^{n} [3.(\deg_G(u_j)+\deg_G(u_k))]$$

ECI(G-++)≤3n(n-1)+3M1(G) , as desired.

Theorem 8.Let G = (n, m) graph. Then

ECI(G+--)≤ 4mn+4m(m+n-1)-4M1(G) (11)

Proof: Let G=(V, E) be a graph with V(G) ={v1, v2, v3, ……., vn} and E(G)={e1, e2, e3, · · ·.. , em}. Then V (G+--)={v1, v2, v3, · · · , vn, e1, e2, e3, · · · , em}.Clearly |V(G+--)| =m+n. further, diam(G+--)≤4.

Since, for every vЄV(G), eG(v)≤ diam(G).Therefore, for every u ∈ G+--, eG+--(u)≤ 4.Let ui ∈ V(G+--) be the corresponding vertex vi ∈ V (G) and uj ∈ V (G+--) be the corresponding vertex ej∈E(G) in G+--.Then degG+--(ui)=m and degG-++(uj)=m+n-1-(deg(vi)-deg(vj)) where ej = vivj. Therefore

$$ECI(G^{+--}) = \sum_{i=1}^{n} (e_{G^{+--}}(u) \deg_{G^{+--}}(u))$$

$$= \sum_{u_i \in V(G^{+--}) \cap V(G)} (e_{G^{+--}}(u_i).\deg_{G^{+--}}(u_i)) + \sum_{u_i \in V(G^{+--}) \cap E(G)} (e_{G^{+--}}(u_j).\deg_{G^{+--}}(u_j))$$

Since eG+--(u) ≤ 4. Therefore,

$$ECI(G^{+--}) \le \sum_{u_i \in V(G)}^{n} [4.(m)] + \sum_{u_j u_k \in E(G)}^{n} [4.(m+n-1-\deg_G(u_j)+\deg_G(u_k)))]$$

ECI(G+--) ≤ 4mn+4m(m+n-1)-4M1(G), as desired.

III. CONCLUSION:

In this paper, we have obtained bounds for all eight transformation graphs of total graph.

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