Performance Analysis and Comparison of Complex LMS, Sign LMS and RLS Algorithms for Speech Enhancement Application

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Abstract—Recent developments in the area of adaptive signal processing have advanced massively due to increase in powerful and cost effective digital signal processors with low cost memory chips. The uses of speech processing system for voice communication and recognition task have become more and more common. These factors lead to promote the use of digital signal processing technology for implementation of emerging applications. The process to remove unwanted interference is common and occurs in many situations. The technique of adaptive filtering is a method by which signal enhancement or noise reduction can be accomplished. An adaptive filter self adjusts its transfer function according to an optimizing algorithm. In this paper we carried out the analysis and experimentation to study the existing adaptive filter algorithms and their application for speech enhancement. The paper describes Least Mean Square (LMS) algorithm and Recursive Least Square (RLS). The complex Least Mean Square (CLMS) algorithm and the modification in CLMS lead to Sign Least Mean Square (SLMS) algorithm. The Sign-Sign Least Mean Square algorithm (SSLMS) is also considered for comparison. Normalization operation is performed on the sample which leads to evolution of NLMS algorithm. The experimentation reveals that LMS have fast convergence than RLS. The computational complexity of RLS is very high as compared to LMS.

Keywords—speech enhancement, adaptive filter, least mean square algorithm and recursive least mean square algorithm

I. INTRODUCTION (HEADING 1)

Adaptive noise cancellation system, an alternative method of estimating signals corrupted by additive noise is described by Widrow et al.[1]. The method uses a primary input containing the corrupted signal and the reference input contains noise correlated in some way with the primary noise. The reference input signal is adaptively filtered and subtracted from the primary input to obtain the signal estimate. Adaptive noise cancellation is a method of optimal filtering that can be applied whenever a suitable reference input is available. The advantage of the method is adaptive capability, low output noise and its low signal distortion. The capability processing the inputs whose properties are unknown and non stationary. It leads to a stable system that automatically turns itself off when no improvement in Signal to Noise Ratio (SNR) can be achieved. Output noise and signal distortion are generally lower than can be achieved with conventional optimal filter configurations.

Speech quality is a measure design aspect in variety of applications like mobile phones, video conferencing, hearing aids and speech recognition system. Significant efforts have been made and many algorithms have been developed in order to reduce the noise level in corrupted speech signal. Noise reduction aims at estimating a desired clean speech signal from its noisy observation. A variety of algorithms that address this problem are available. The technique of adaptive filtering is one medium by which signal enhancement or noise reduction is accomplished. In a similar adaptive fashion, systems submerged in an unknown environment can be detected with a system identification structure. An adaptive filter is a filter that self-adjusts its transfer function according to an optimizing algorithm. A extensive list of noise procedures have been defined to measure noise in signal processing in absolute terms, relative to some standard noise level, or relative to the desired signal level like dynamic range, defined by inherent noise level, Signal-to-Noise ratio (SNR), ratio of noise power to signal power, Peak Signal-to-Noise Ratio (PSNR), maximum SNR in a system.
The structure that defines how the output signal of the filter is computed from its input signal.

The parameters within this structure that can be iteratively changed to alter the filter input output relationship

The adaptive algorithm that describes how the parameters are adjusted from one time instance to the next.

By choosing a particular adaptive filter structure, it specifies the number and type of parameters that can be adjusted. The adaptive algorithm is used to update the parameter values of the system to optimize mean square error value or useful task at hand.

### A. Adaptive Algorithms

An adaptive algorithm is a procedure for adjusting the parameters of an adaptive filter to minimize a cost function chosen for a task at hand. In practice quantity of interest is not always \( d[n] \). There are situations in which \( d[n] \) is not available at all times, in such cases adaptation occurs only when \( d[n] \) is available. When \( d[n] \) is unavailable, the most recent parameter estimates are used to compute \( y[n] \). There are real world situations in which \( d[n] \) is never available then, a hypothetical \( d[n] \) is predicted in applications like blind adaptive algorithm. The relation between \( x[n] \) and \( d[n] \) varies with time in such situations adaptive filter attempts to alter its parameters to follow the changes in this relationship as encoded by \( x[n] \) and \( d[n] \) this behavior is called tracking.

Over the last decade there has been significant research directed towards development of adaptive algorithms. There are basic two algorithms, the Least Mean Square (LMS) algorithm, which is based on a gradient optimization for determining the coefficients and the class of recursive least squares algorithms, which includes both direct form FIR and lattice realization [2].

1. IMPLEMENTATION OF LEAST MEAN SQUARE ALGORITHM

When the adaptive filter has a tapped delay line FIR structure, then the LMS update algorithm is simple. Typically, after each sample, the coefficients of the FIR filter are adjusted with respect to weight update equation. The algorithm here does not require the input values to have any particular relationship; therefore it can be used to adapt a linear FIR filter. The update equation is given as:

\[
w(n+1) = w(n) + \mu e(n)w(n)
\]

Where \( \mu \) is the convergence parameter (i.e. step-size), \( e(n) = d(n) - w^T(n)x(n) \) is the output error, \( d(n) \) is the desired signal and \( x(n) \) is the input signal.

The effect of the LMS algorithm is at each time \( n \), to make a small change in each weight. The direction of the change is such that it would decrease the error if it has been applied at time \( n \). The magnitude of the change in each weight depends on \( \mu \), the associated input value and the error at time \( n \). The weights making the largest contribution to the output are changed the most. If the error is zero, then there should be no change in the weights. If the associated value of inputs to algorithm is zero, then changing the weight makes no difference, so it is not changed.

Convergence factor \( \mu \) controls how fast and how well the algorithm converges to the optimum filter coefficients. If \( \mu \) is too large, the algorithm will not converge. If \( \mu \) is too small the algorithm converges slowly and may not be able to track changing conditions. If \( \mu \) is large but not too large to prevent convergence, the algorithm reaches steady state rapidly but continuously overshoots the optimum weight vector. Sometimes, \( \mu \) is made large at first for rapid convergence and then decreased to minimize overshoot.

III. IMPLEMENTATION OF RECURSIVE LEAST SQUARE ALGORITHM (RLS)

Recursive Least Square is an algorithm that finds filter coefficients to minimize weighted linear square cost function related to input signal, whereas LMS aims to reduce mean square error.

In derivation form RLS is deterministic (i.e. has some specific equation), whereas LMS & other algorithms are stochastic (i.e. non-deterministic/random in nature).

The RLS algorithm depends on forgetting factor \( \lambda \) (lambda). The smaller \( \lambda \) indicates smaller contribution of previous values. So filter becomes more sensitive to recent samples hence more fluctuations in filter coefficients. \( \lambda \) is hence taken between 0.98 & 1. That is it indicates how quickly filter forgets past sample value.

When \( \lambda \) is close to 1, algorithm achieves low misadjustment & good stability, but tracking capabilities are reduced. When \( \lambda \) is smaller, it improves tracking but increase misadjustment & affects stability of algorithm.

By the use of this algorithm equation a recursive loop have been introduced. Thus set certain predefined constant values, and then the equation can be updated. The algorithm equations are adjusted by initially considering a higher value and then
taking its inverse. For that purpose forgetting factor is used. Forgetting factor will decide the step size as well as convergence speed of the filter. The weight update equation for RLS algorithm is given by,

\[ z(n) = w(n-1) \cdot y(n)^T \]  
\[ e(n) = d(n) - z(n) \]  
\[ k(n) = \frac{P(n-1) \cdot z(n)}{\lambda + z(n)^H \cdot P(n-1) \cdot z(n)} \]  
\[ P(n) = \frac{P(n-1) - P(n-1) \cdot z(n) \cdot z(n)^H \cdot P(n-1)}{\lambda} \]  
\[ w(n) = w(n-1) + e(n) \cdot k(n) \]

A. COMPARISON BETWEEN LEAST MEAN SQUARE AND RECURSIVE LEAST SQUARE

- In the LMS algorithm, the correction that is applied in updating the old estimate of the coefficient vector is based on the instantaneous sample value of the tap-input vector and the error signal. On the other hand, in the RLS algorithm the computation of this correction utilizes all the past available information.
- The LMS algorithm requires approximately 20M iterations to converge in mean square, whereas RLS converges in mean square within less than 2M iterations. The rate of convergence of the RLS algorithm is therefore, in general, faster than that of the LMS algorithm by an order of magnitude.

Unlike the LMS algorithm, there are no approximations made in the derivation of the RLS algorithm. Accordingly, as the number of iterations approaches infinity, the least squares estimate of the coefficient vector approaches the optimum Wiener value, and correspondingly, the mean square error approaches the minimum value possible. In other words, the RLS algorithm, in theory, exhibits zero misadjustment. On the other hand, the LMS algorithm always exhibits a nonzero maladjustment; however, this misadjustment may be made arbitrarily small by using a sufficiently small step-size parameter \( \mu \).

A great deal of research efforts have been used up characterizing the role that \( \mu(n) \) plays in the performance of adaptive filter in terms of statistical or frequency characteristics of the input and desired response signal. Often success or failure of an adaptive filtering application depends on how the value of \( \mu(n) \) is chosen or calculated to obtain the best performance from the adaptive filter. The issue of choosing \( \mu(n) \) for both stable and accurate convergence of LMS is very important. LMS incorporates an iterative formula that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which finally leads to the minimum mean square error. Compared to other algorithms LMS algorithm is relatively simple; it does not require correlation function calculation nor does it require matrix inversions.

**Parameters:**
- \( M \) = number of taps (filter length)
- \( \mu \) = step size parameter \( 0 < \mu \)

The Normalized LMS (NLMS) introduces a variable adaptation rate. It improves the convergence speed in a non-stationary environment. In another version, the Newton LMS, the weight update equation includes whitening in order to achieve a single mode of convergence. For long adaptation processes the Block LMS (BLMS) is used to make the LMS faster. In Block LMS (BLMS), the input signal is divided into blocks and weights are updated block wise. A simple version of LMS is called the Sign LMS (SLMS). It uses the sign of the error to update the weights. Also, LMS is not a blind algorithm i.e. it requires a priori information for the reference signal.

**Parameters:**
- \( M \) = number of taps (filter length)
- \( \mu \) = step size parameter \( 0 < \mu \)

Where,
- \( E \left[ |e(n)|^2 \right] \) = error signal power
- \( E \left[ |u(n)|^2 \right] \) = input signal power
- \( D(n) \) = mean square division

**Initialization:**
- If prior knowledge about the tap-weight vector \( w(n) \) is available, use the knowledge to select an appropriate value for \( \hat{w}(0) \). Otherwise set \( \hat{w}(0) = 0 \).

**Data Given:**
- \( u(n) \) = M-by-1 tap input vector at time n.
- \( d(n) \) = desired response at time step n. To be computed: \( \hat{w}(n+1) \) = estimate of tap-weight vector at time step n+1

**Computation:**
- For \( n = 0, 1, 2, \ldots \) compute \( e(n) = d(n) - \hat{w}(n)u(n) \cdot \hat{w}(n+1) \).

B. Convergence Issues in the LMS Adaptive Filters

Least Mean Square (LMS) adaptive filter is the most popular algorithm and widely used adaptive system, appearing in numerous commercial and scientific applications.

The LMS adaptive filter is described by the mathematical equation as

\[ W(n + 1) = W(n) + \mu(n)X(n) \]  
\[ e(n) = d(n) - W(n)X(n) \]
Where

\[ W(n) = [w_1(n) \ldots w_L(n)] \]

is the coefficient vector.

\[ X(n) = [x(n)x(n-1) \ldots x(n-L+1)]^T \]

is the input signal vector.

\( d(n) \) is the desired signal.

\( e(n) \) is the error signal.

\( \mu(n) \) is the step size.

There are three main reasons for the popularity of LMS adaptive filter. First, it is relatively easy to implement in software and hardware due to its computational simplicity and efficient use of memory. Secondly, it performs robustly in the presence of numerical errors caused by finite-precision arithmetic. Third, its behavior has been analytically characterized up to such an extent that users can setup systems to obtain adequate performance with only limited knowledge about the input and the desired response.

C. Performance Comparison Of Least Mean Square Normalization LMS

The experimentations were carried out for LMS algorithm in airport noise with different level of noise 0 dB, 5 dB and 10 dB. Figure 1 show the detail results of the same. The Performance comparison of speech enhancement using NLMS algorithm in presence of airport noise is shown in figure 2.

### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LMS</th>
<th>NLMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>Good</td>
<td>At par (High value for less noisy data)</td>
</tr>
<tr>
<td>MSE</td>
<td>At par</td>
<td>More error</td>
</tr>
<tr>
<td>Execution Time</td>
<td>Converges fast</td>
<td>Converges very slow</td>
</tr>
</tbody>
</table>

The results show that SNR performance of LMS is good while NLMS performs at par, have higher value of SNR for less noisy data. The MSE performance of LMS is better than NLMS. In case of NLMS the MSE value is high for high noise level and gives low MSE for low level of noise.

D. RECURSIVE LEAST SQUARE (RLS)

Recursive Least Square algorithm (RLS) is another important class of algorithm. The central problem in estimation is to recover, with good accuracy, a set of unobservable parameters from corrupted data. Several optimization criteria have been used for estimation purpose over years, but the most important is based on the quadratic
The most important is linear least square criterion, which was developed by Gauss [14].

The properties of least square solution is it can be evaluated in closed forms, it can be recursively updated as more input data are made available and are maximum likelihood estimators in the presence of Gaussian measurement noise. Thus by setting some predefined initial constant values, the algorithm can be updated. By considering some higher values and then taking its inverse this algorithm can be adjusted, for this purpose forgetting factor is used. The forgetting factor step size as well as convergence speed of the adaptive filter. The weight update equation for RLS algorithm is expressed as,

\[ z(n) = w(n-1)y^T(n) \]  
\[ e(n) = d(n) - z(n) \]  
\[ k(n) = \frac{P(n-1) \cdot z(n)}{\lambda + z^H(n) \cdot P(n-1) \cdot z(n)} \]  
\[ P(n) = P(n-1) - P(n-1) \cdot z^H(n) \cdot k(n) \]  
\[ w(n) = w(n-1) + e(n) \cdot k(n) \]  

Table 1.1 Results for LMS algorithm

<table>
<thead>
<tr>
<th>Algorithm Method: LMS</th>
<th>Type of Noise: Airport Noise</th>
<th>Noise level</th>
<th>SNR</th>
<th>MSE</th>
<th>TIME</th>
<th>ID: 853</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0dB</td>
<td>11.1708</td>
<td>0.000218</td>
<td>2.39115</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5dB</td>
<td>9.9084</td>
<td>0.000193</td>
<td>0.24397</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10dB</td>
<td>9.1533</td>
<td>0.000186</td>
<td>0.24631</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.2 Results for RLS algorithm

<table>
<thead>
<tr>
<th>Algorithm Method: RLS</th>
<th>Type of Noise: Airport Noise</th>
<th>Noise level</th>
<th>SNR</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0dB</td>
<td>24.7705</td>
<td>1.14E-05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5dB</td>
<td>22.6959</td>
<td>1.27E-05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10dB</td>
<td>21.6435</td>
<td>1.35E-05</td>
</tr>
</tbody>
</table>

The table 1.1 and 1.2 lists the results of LMS algorithms and RLS algorithm. It is observed that the range of SNR is 9.15 to 11.17 dB with MSE 0.000186 to 0.000218. The execution time is 0.24 to 2.3 s. The RLS provide much better SNR ranging from 21.6 to 24.77 dB with better MSE in range of 1.14E-05 to 1.35E-05 with slightly more execution time.

E. CONCLUSION

This paper discussed the implementation of existing adaptive filter algorithms like Least Mean Square and Recursive Least Square. Researchers have made certain modification in LMS which leads to evolution of Leaky LMS, CLMS, SLMS and SLMS. The performance of these algorithms is compared on the basis of the performance indices like signal to noise ratio, MSE and Convergence speed.

The experimentation reveals that LMS have fast convergence than RLS. The computational complexity of RLS is very high as compared to LMS and performs well in speech enhancement application.

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